POWER SYSTEM VOLTAGE STABILITY: A SHORT TUTORIAL

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(The demos included and the material in part are provided by Dr. Thierry Van Cutsem)

Power system stability and voltage stability.

☐ Is power system stability a single problem?

YES!

"is the property of a power system which enables it to remain in a state of equilibrium under normal operating conditions and to regain an acceptable state of equilibrium after a disturbance"

☐ What is voltage stability?

"voltage instability stems from the attempt of load dynamics to restore power consumption beyond the capability of the combined transmission and generation system"

☐ What is voltage collapse?

"the process by which the sequence of events accompanying voltage instability leads to a low unacceptable voltage profile in a significant part of the power system"

(may or may not be the final outcome of voltage instability)

Voltage stability and angle stability.

- Is there clear distinction between angle and voltage instability?

NOT ALWAYS!

“often both types of instabilities come together and one may lead to other”

- Distinction between the two types is important for understanding of the underlying causes of the problems in order to develop appropriate design and operating procedures.
- Distinction is effective but the overall stability of the system should be kept in mind.
- Solutions for one problem should not be at expense of another.
- It is essential to look at all aspects of the stability phenomena and at each aspect from more than one viewpoint.

Why Voltage Stability is more and more important?

- Generation centralized in fewer, larger power plants:
  - fewer voltage controlled buses
  - longer electrical distances between generation and load
- Extensive use of shunt capacitor compensation
- Voltage instability caused by line and generator outages
- Many incidents throughout the world (France, Belgium, Sweden, Japan, USA, etc.)
- Operation of system closer to its limits

Main results of the post-mortem analysis of 20 incidents

☐ The initial event may be due to a variety of causes:
- Small gradual changes such as natural increase in system load,
- Large sudden disturbances such as loss of a generating unit or a heavily loaded line,
- cascading events

☐ The inability of the system to meet its reactive demands.

☐ Voltage collapse generally manifests itself as a slow decay in voltage.

☐ Voltage collapse is strongly influenced by system conditions and characteristics:
- large distances between generation and load,
- ULTC actions,
- Unfavorable load characteristics,
- Poor coordination between various control and protective systems

☐ The voltage collapse may be aggravated by excessive use of shunt capacitor compensation.

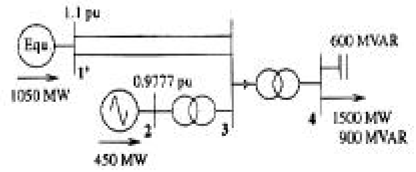
Voltage stability analysis

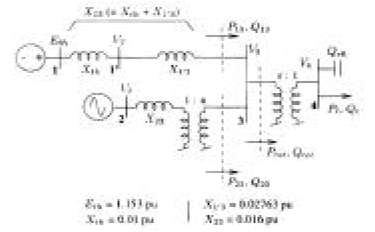
☐ Mechanism of voltage instability:
- How and why does instability occur?
- What are the key contributing factors?
- What are voltage-weak areas?
- How to improve voltage stability ?

☐ Proximity to voltage instability:
- How close is the system to voltage instability?

{Closely related issue: voltage (in)stability indices (indicators)}

Voltage instability illustration on a simple system.





```
BUS  1  380.00    0.0    0.0    0.0    0.0  ;
BUS  2   20.00    0.0    0.0    0.0    0.0  ;
BUS  3  380.00    1500.0   300.0    0.0    0.0  ;
LINE '1-3' 1 3
0. 79.8  0. 1350. 1  ;
LINE '1-3b' 1 3
0. 79.8  0. 1350. 1  ;
TRFO '2-3' 2 3 ' '
0.0  8.  0.0  104.0  500.0  0. 0. 0  0. 0.  1 ;
GENER 1 1 1
1050.0  0.0  1.1  900.0  -9999.  9999.  1  ;
GENER 2 2 2
300.0  0.0  0.9777  500.0  -9999.  9999.  1  ;
SLACK 1 ;
GROUP-PV '1' 1. ;
GROUP3 '2'
2.1  2.1  0.1  0.  0.  50.  0.  0. ;
OXL '2'
0.  2.825  20.  20. ;
LTCDYN '4-3'
20. 10. ;
LOAD(V)* 2. 2. ;
```

Voltage instability in a simple system – modelling 1.

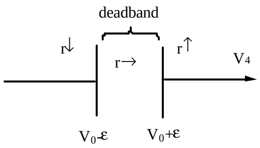
Load modeling:

$$P_l = P_0\left(\frac{V_4}{V_0}\right)^a \quad Q_l = Q_0\left(\frac{V_4}{V_0}\right)^b$$

and including the shunt:

$$Q_{tot} = (Q_0 - Q_{sh})\left(\frac{V_4}{V_0}\right)^b$$

The load tap changer:



$$t_{in} = 20s \quad t_{ul} = 10s \quad [0.99,1.01]pu$$

Maximum deliverable power – 1.

Thinking on:

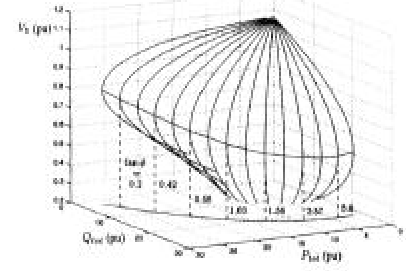
$Z_{load} = Z^*_{sys}$ Is not realistic.

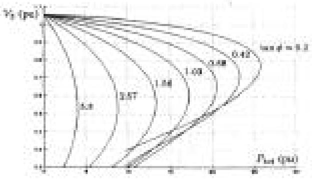
Realistic: maximum deliverable power keeping power factor constant.

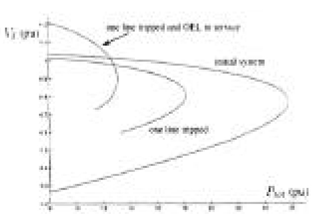
For the simple system considered:

$$P_{tot} = P_{23} \pm \frac{E_{th}V_3}{X_{13}}\sqrt{1 - \frac{X_{13}^2}{E_{th}^2V_3^2}\left(Q_{tot} + V_3^2\,(\frac{1}{X_{13}} + \frac{1}{n^2X_{23}} - \sqrt{\frac{V_2^2V_3^2}{n^2X_{23}^2} - P_{23}^2}\right)^2}$$

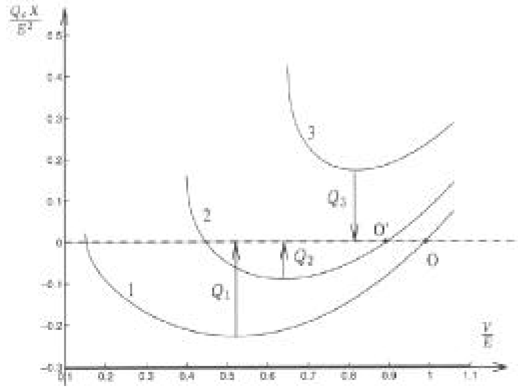
Maximum deliverable power - 2.







V-Q curves.

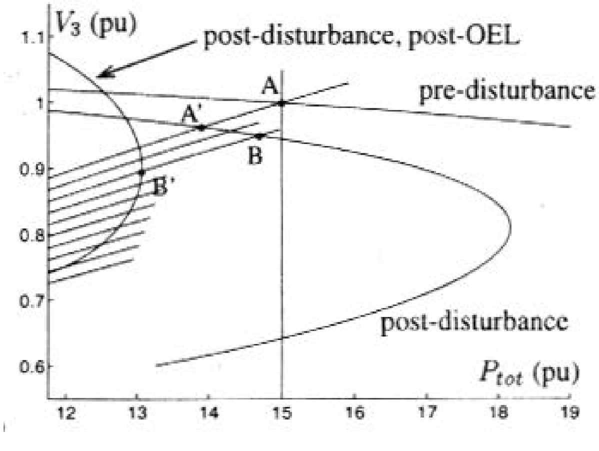


V-Q curve is a characteristic of both the network and the load

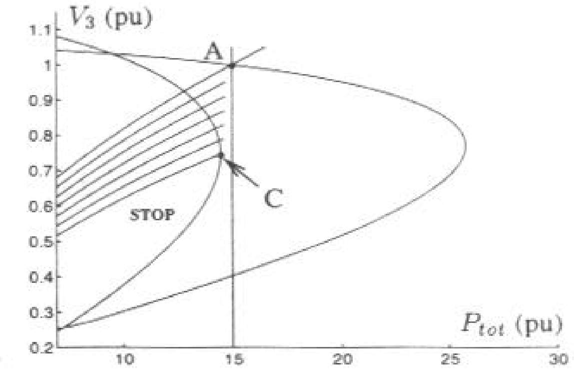
Q1 and Q2 are reactive power margins with respect to
The loss of operating point.

Q3 provides a measure of the Mvar distance to system
Operability.

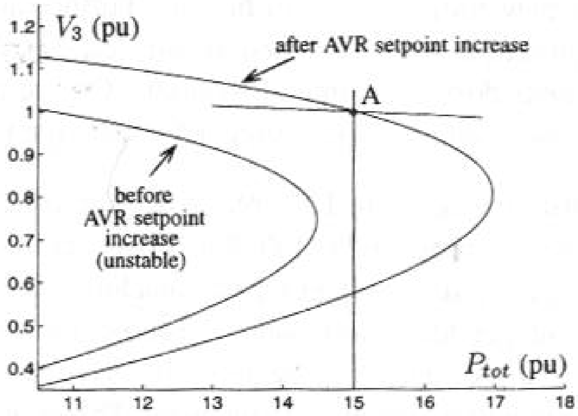
Example 1: loss of a line with LTC-controlled load.



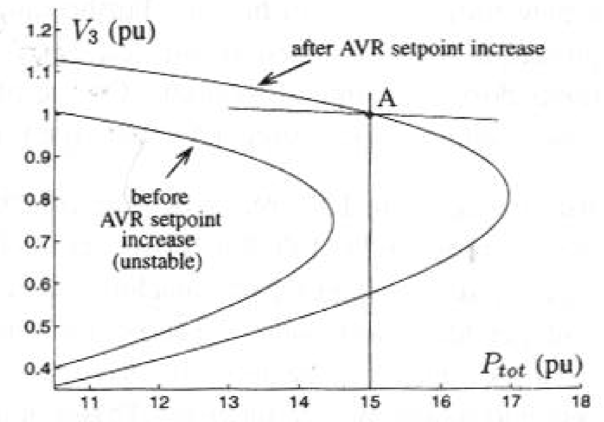
Example 2: Corrective controls – LTC blocking.



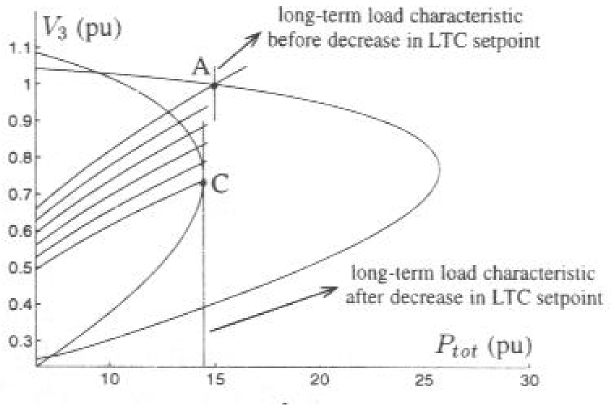
Example 3: Corrective controls – capacitor switching.



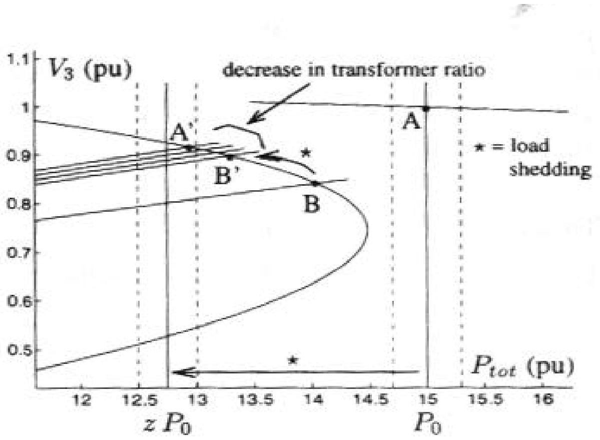
Example 4: Corrective controls – generator voltage increase.



Example 5: Corrective controls – LTC set-point decrease.



Example 6: Corrective controls – load shedding.



From dynamic model to load flow Jacobian.

Power system dynamics is naturally described by differential-algebraic equations:

$$\begin{aligned}\dot{x} &= f(x,y,m)\\ 0 &= g(x,y,m)\end{aligned} \quad \text{or} \quad \begin{bmatrix}\dot{x}\\ 0\end{bmatrix} = F(z,m)$$

Assuming Jacobian $D_y g(\bullet)$ is nonsingular: $\dot{x} = f(x, y^{-1}(x,m), m) = s(x,m)$

An equilibrium point: (z_0, m_0) is defined by: $F(z_0, m_0)$

An equilibrium point: (z_*, m_*) where: $D_z F(z_*, m_*)$ Singular bifurcation Point.

IMORTANT: There is direct relation between singularities of the power flow Jacobian and actual bifurcations of the full dynamical system

Beyond ordinary (vanilla) power flows.

- Distributed “slack” bus.
- Q limits:
  - PV→ PQ
  - Active power and Voltage dependent

$$Q_{\lim} = a_1P^2 + a_2V^2 + a_3PV + a_4P + a_5V + a_6$$

- Active power limits.
- Export flows.
- Line overloads.

Continuation Power Flows

Effective continuation method, and consequently continuation power flow, solves the problem via four basic elements:

- Predictor. Its purpose is to find an approximation for the next solution. Usually tangent, first-order polynomial, or zero order polynomial (as it is the case in this paper) predictor is employed.
- Parameterization. Mathematical way of identifying each solution on the solution curve. Parameterization augments the system of power flow equations.
- Corrector. Usually, application of Newton method to the augmented system of equations.
- Step length control. Can be done by optimal fixed step length or by adaptive step length control.

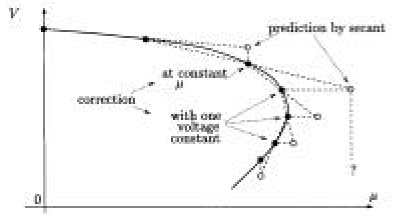
Continuation Power Flows

Typical natural parameters of interest include the following:

- ☐ The total system demand.
- ☐ The demand at a given bus or within a given area.
- ☐ The amount of power transfer between two areas or between two buses.
- ☐ Some other parameters such as the impedance of a line, etc.

CPF – Mathematical formulation - 1

$$f(x,m)=0$$



$$P=P_0\left(\frac{V}{V_0}\right)^a \quad ; \quad Q=Q_0\left(\frac{V}{V_0}\right)^b$$

$$P_{Li}=(1+k_{Li}m)P_{Li0}\left(\frac{V_i}{V_{i0}}\right)^{ai}$$

$$Q_{Li}=(1+k_{Li}m)Q_{Li0}\left(\frac{V_i}{V_{i0}}\right)^{bi}$$

$$\Delta P_{total}=\sum_{i=1}^{n}P_{Li}-P_{total\,0}$$

$$P_{Gi}=P_{Gi0}+k_{Gi}\Delta P_{total}$$

$$\Delta P_i=P_{Gi0}+k_{Gi}\left(\sum_{i=1}^{n}P_{Li}-P_{total\,0}\right)-P_{Li}-P_{Ti}$$

$$\Delta Q_i=Q_{Gi0}-Q_{Li}-Q_{Ti}$$

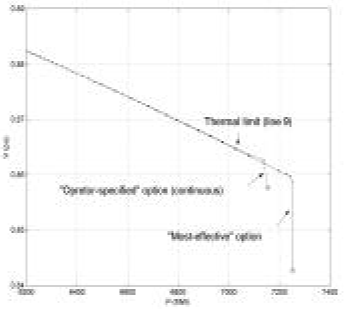
CPF – Mathematical formulation - 2

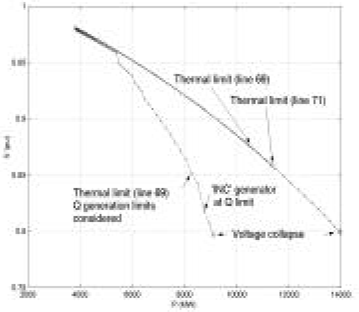
$$\begin{bmatrix}\Delta P\\ \Delta Q\\ \hline rhs\end{bmatrix} = \left[\begin{array}{cc|c} H & N & \\ M & L & k1\\ \hline \multicolumn{2}{c|}{FF1} & kF1\end{array}\right] \times \begin{bmatrix}\Delta q\\ \Delta V\\ \hline \Delta l\end{bmatrix}$$

Continuation power flow with overload and generation re-dispatch

$$\begin{bmatrix}\Delta P\\ \Delta Q\\ \hline rhs1\\ \hline rhs2\\ \hline rhs3\end{bmatrix} = \left[\begin{array}{cc|c|c|c} H & N & k1 & & \\ M & L & & k2 & \\ \hline \multicolumn{2}{c|}{FF1} & kF1 & & k3\\ \hline \multicolumn{3}{c|}{FF2} & 0 & \\ \hline \multicolumn{4}{c|}{FF3} & 0\end{array}\right] \times \begin{bmatrix}\Delta q\\ \Delta V\\ \hline \Delta l\\ \hline FL_1\\ \hline FL_2\end{bmatrix}$$

CPF – Example using IEEE-118 and 39 test systems.

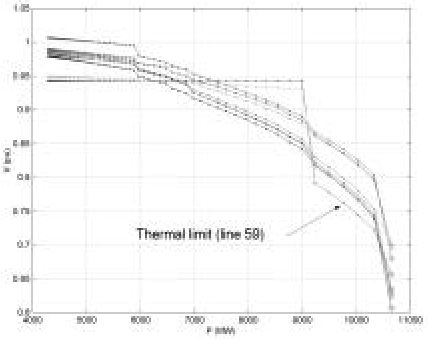




IEEE – 39 test system:
- Without limits 11998.54 MW
- with flow limits 7245 MW

IEEE – 118 test system:
Evolution of voltage at the system critical
Bus (bus 95)

CPF – Example using IEEE-118 test system.



IEEE – 118 test system:
-all limits considered
-Without flow limits max. loadabality 19250.38 MW
-With flow limits max. loadability 10156.92 MW

Voltage Stability Indices (indicators)

Sensitivity factors:

$$f(x,m)=0 \qquad SF = \left\| \frac{dx}{dm} \right\|$$

$$VSF = \left\| \frac{dV}{dm} \right\|$$

- -Loading margin,
- -Local load margins,
- -Reduced determinant,
- -Tangent vector index,
- -System determinant,
- -Reactive power margins,
- -V/V_0 index.

Singular values and Eigenvalues:

$$J = W \Lambda U^T = \sum_{i=1}^{n} w_i m_i v_i^T$$

W – complex matrix of left eigenvectors
U – complex matrix of right eigenvectors
Λ - diagonal matrix of complex eigenvectors

Voltage Stability Indices (indicators)

V-Q sensitivities:

$$\begin{bmatrix}\Delta P\\ \Delta Q\end{bmatrix}=\begin{bmatrix}J_{Pq} & J_{PV}\\ J_{Qq} & J_{QV}\end{bmatrix}\begin{bmatrix}\Delta q\\ \Delta V\end{bmatrix}$$ by letting: $\Delta P = 0$

$\Delta Q = J_R \Delta V$ where: $J_R = \left[J_{QV} - J_{Qq} J_{Pq}^{-1} J_{PV}\right]$

$$\left(\frac{dV_i / V_i}{dQ_{Gj} / Q_{Gj}}\right)_{P_{Li}=P_{Li0}} > 0 \quad i \in a_L,\, j \in a_G$$

Also possible: $\left(\frac{dQ_{Li}}{dQ_{Gj}}\right)_{P_{Li}=P_{Li0}} > 0 \quad i \in a_L,\, j \in a_G$

Voltage Stability Indices (indicators)

Also possible: $z_R = \frac{\sum_{i\in R} Q_{li}(d\overline{Q}_G / dQ_{Li})}{\sum_{i\in R} Q_{Li}}$

"R" is a region of the system of particular interest

BPA Voltage Stability Index:

$Index = 100\left[1 - \frac{\Sigma W_i Q_i}{\Sigma W_i Q_{\max i}}\right]$ where: $W_i = \frac{\Delta Q_i}{\Sigma \Delta Q_i}$

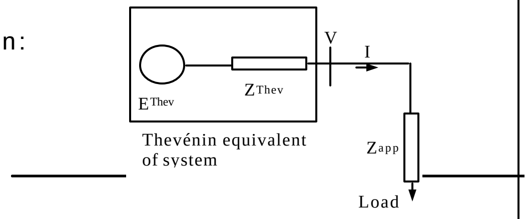
and can be normalized as: $W_i' = \frac{\Delta Q_i}{\Delta Q_{\max i}}$

Voltage Stability Indices (VIP – voltage instability predictor)

Maximum transfer occurs when:

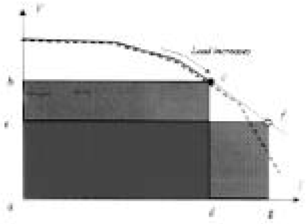
$$|\overline{Z}_{app}| = |\overline{Z}_{Thev}|$$

$$\overline{Z}_{app} = \frac{\overline{V}}{\overline{I}}$$



Margin can be expressed in MVA:

$$\Delta S = \frac{(V_t - Z_{Thev} I_t)^2}{4 Z_{Thev}}$$



Static vs. Time Domain Methods

Static methods:

- Capture the loss of a long-term equilibrium,
- Based on algebraic equations that stem from the equilibrium conditions of long-term dynamics,
- Cannot (easily) account for controls that depend on the system time evolution

Time domain methods:

- Higher modeling accuracy,
- Possibility to study other instability mechanisms than the loss of equilibrium,
- Higher interpretability of results.

Approximation of long-term equilibrium equations by standard load flow equations

Standard Load flow	True long-term equilibrium calculation
loads	
Constant power	If controlled by LTC: -if LTC not limited: constant power -If LTC limited: consider short-term characteristics Load self-restoration (consider long-term char.) Other cases (consider short-term)
generators	
Constant voltage Constant reactive power	Under voltage control: -voltage drop effect Under rotor current limit: -reactive power output varies with voltage and active power
- Active power imbalance not left to slack-bus but shared by generators according to governor/LFC effects - Update reactive power capability with active power output	

Quasi Steady_State (QSS) Long-Term Simulation

Load evolution $w = f(t)$

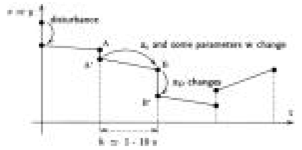
Generic model of load self-restorat.
Secondary voltage control

$$\dot{z}_C = h_C(x, y, z_C, z_D, w)$$

Load frequency control
Load tap changers
Overexcitation limiters
Automatically switched capacitors

$$z_D(k+1) = h_D(x, y, z_C, z_D(k), w)$$

Generators and regulators
SVCs
HVDC
Induction motors



Long-Term dynamics

Short-term dynamics replaced
By equilibrium equations

QSS main features

- ❑Compromise between efficiency of static methods and advantages of time-domain methods,
- ❑Accurate enough for security analysis,
- ❑Adequate for real-time applications
- ❑Focuses on long-term dynamics
- ❑Cannot deal with severe disturbances to short-term instability (A solution: couple transient stability simulation and QSS)
- ❑Cannot simulate the final collapse in cases where short-term dynamics becomes unstable due to long-term instability (usually not of interest in security analysis)
- ❑Loss of short-term equilibrium can be detected by QSS method

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QSS main features - 2

Power Utilities and Companies using ASTRE:

- ❑EDF (France),
- ❑Hydro-Quebec (Canada),
- ❑ELIA (Belgium),
- ❑HTSO (Greece)
- ❑CESI, GRTN (Italy, within the OMASES project)

Voltage security analysis

ENDS	MEANS
Evaluate impact of contingencies	Post-contingency load flow Modified load flow VQ curves Multi-time-scale simulation QSS long-term simulation
Determine maximum stress allowed for the system (loadability limit)	Continuation power flow Optimization methods Time simulation coupled with sensitivity analysis
Combine contingency and stress analysis	Post-contingency loadability limit Secure operation limit
Preventive or corrective control	Sensitivity & eigenvector based methods Optimal power flow

Post-contingency load flow

Evaluate the impact of contingency by computing the post-contingency long-term equilibrium,

In unstable cases with no long-term equilibrium, any numerical method trying to solve the equilibrium equations will diverge

Simple instability indicator, but:

- ☐Divergence may be caused by pure numerical problems
- ☐In truly unstable case, following divergence, we are left without
- ☐Information on the nature and location of the problem, remedies, etc.

An example of Modified Load Flow

$g(y) = 0$ y – state vector; g – power “mismatch” vector

Usual Newton-Raphson iterations (k=1,2,...):

solve $g_y \Delta y = -g(y^{k-1})$

increment $y^k = y^{k-1} + \Delta y$

Non divergent load flow:

solve $g_y \Delta y = -g(y^{k-1})$

increment $y^k = y^{k-1} + a^* \Delta y$

where a^* minimizes $\sum_i g_i^2 (y^{k-1} + a \Delta y) = F(a)$

All mismatches negligible: solved

No solution:

The method provides a solution with minimum mismatches.
The largest mismatches point out the voltage problem area indicating where power should be injected to restore solvability

CAUTION: Generator limits handling require great care.

Optimization methods

Solve: $\max \quad m$

$s.t. \quad j\,(u, p^0 + md) = 0$

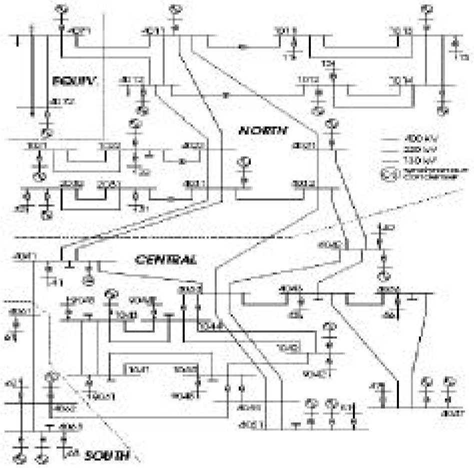
(Necessary) optimality conditions: $L = m + w^T j\,(u, p^0 + md)$

$$\frac{\partial L}{\partial u} = 0 \quad \frac{\partial L}{\partial w} = 0 \quad \frac{\partial L}{\partial m} = 0$$

With respect to continuation power flow:

- faster
- cannot (at least easily) incorporate controls that depends on the system “path” between base case and max. loadability (bifurcation) points

Examples using QSS simulation: Nordic – 32 test system.



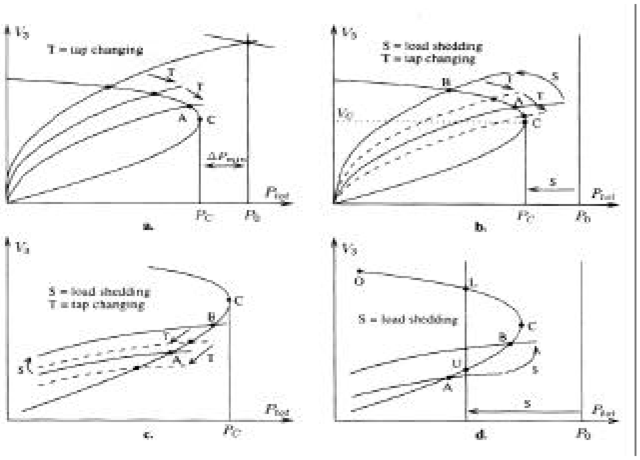
A slightly modified version of a test system used by CIGRE Task Force 38.02.08.

All the loads in the system modeled as voltage dependent (exponential model with 1 for active power and 2 for reactive power.)

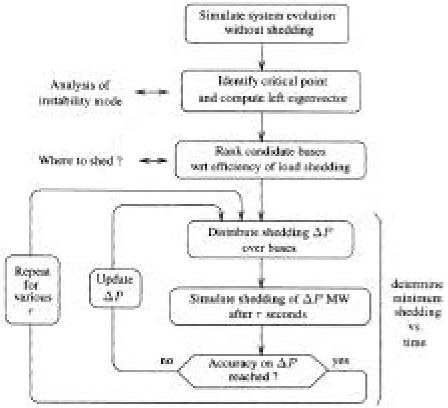
All the loads, except in the equivalent area are controlled By LTCs.

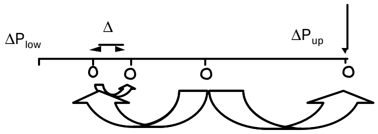
All generators protected by OXL with fixed time delay.

Optimal load shedding: the time vs. amount issue



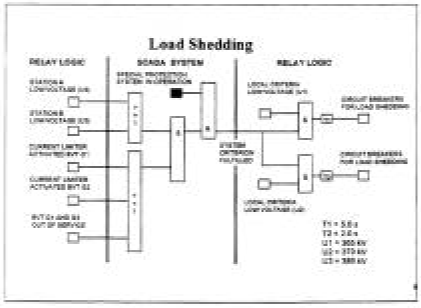
Load shedding: analysis procedure





Binary search

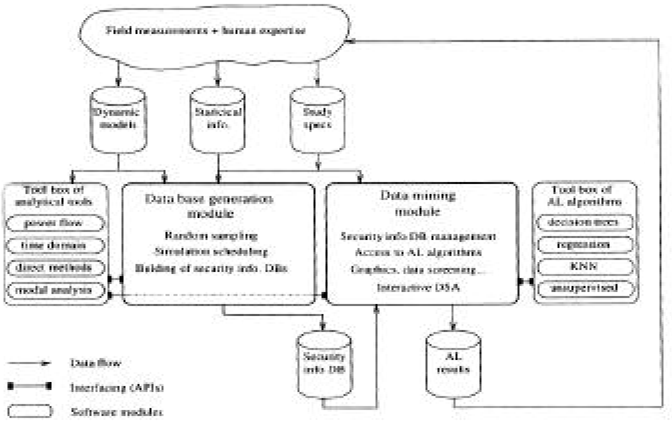
System protection schemes against voltage collapse



Example of SPS against voltage collapse in the South part of Sweden

Recent research conducted at University of Liege: Closed-loop SPS (load shedding)

Automatic learning techniques for voltage security assessment.



Automatic Learning – main steps.

- ❑Building of a large data base of pre-analyzed scenarios (off-line).
- ❑Applying AL methods to extract the complex relationships between the attributes and the classification or margin (off-line).
- ❑On-line. The obtained synthetic information is very simple and fast to use.

Useful links.

www.pserc.wisc.edu (useful papers and reports)

http://power.ece.drexel.edu/ (free software, Matlab,...)

http://thunderbox.uwaterloo.ca/~claudio/claudio.html (free software, Matlab, C/C++, IEEE Report on Voltage stability)

http://www.engr.wisc.edu/ece/faculty/ (follow web pages of Prof. Alvarado, Dobson, and DeMarco)

www.montefiore.ulg.ac.be

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