Linear Control Systems
General Informations

Guillaume Drion

Academic year 2020-2021

SYST0003 - General informations

- **Website**: http://sites.google.com/site/gdrion25/teaching/syst0003
- **Contacts**: Guillaume Drion - forum eCampus
- **Organization**: Virtual (Pre-recorded videos on eCampus, Q/A sessions)
- The course follows the resources provided on the website: http://www.cds.caltech.edu/~murray/amwiki/index.php/Main_Page
- Slides and other files will be posted on the main website.

Two versions of the course

- **SYST0003-1 (5 ECTS): all but electrical engineers**
  - Two projects: one on time domain design, and one on frequency domain design.
- **SYST0003-2 (3 ECTS): electrical engineers**
  - Combined with major project (APRI0007)
  - One project: PID design using frequency domain methods for APRI0007

Goals of the course and evaluation

- **Goals of the course:**
  - Lessons: theory and **intuition**! The main goal of this course is to provide a general (and simple) framework for the design of control systems.
  - Projects: to grasp and apply the concepts of the course.

- **Evaluation:**
  - Projects (personal) - 60% (30% each): based on reports.
  - Written exam - 40%: theory and concepts.

Textbook

- The course follows the textbook freely available at http://www.cds.caltech.edu/~murray/amwiki/index.php/Main_Page
- The chapters corresponding to the videos are given in the online suggested schedule

Suggested schedule

- **Videos will arrive in two "blocs":** one for the time domain part, and one for the frequency domain part. You will be free to use them to help for your project, but to help you organize your work, a suggested schedule is provided on the website:

Schedule

Week	Date		
1	18/09	Lesson #1: Introduction, feedback principles Chapters 1 and 2.	
2	25/09	State feedback - Chapter 7.	
3	02/10	Output feedback - Chapter 8.	
4	09/10	Homework I - Session I.	
5	16/10	Homework I - Session II.	
6	23/10	Frequency domain analysis - Chapter 10.	
7	30/10	PID control - Chapter 11.	
8	06/11	-	
9	13/11	Frequency domain design - Chapter 12.	
10	20/11	Homework II - Session I	
11	27/11	Homework II - Session II	
12	04/12	Fundamental limits - Chapter 14.	
13	11/12	-	-
14	18/12	-	-

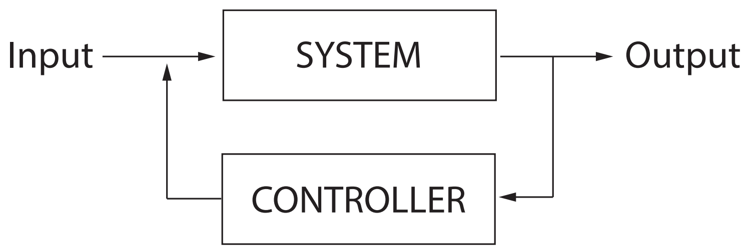
Linear Control Systems
Lecture #1 - Introduction to control systems

Guillaume Drion
Academic year 2020-2021

Systems modeling in three courses

- SYST0002: Modeling and analysis of systems: **open loop**.
  "Observing and analyzing the environment"

- SYST0003: Linear control systems: **closed loop**.
  "Interacting with the environment"

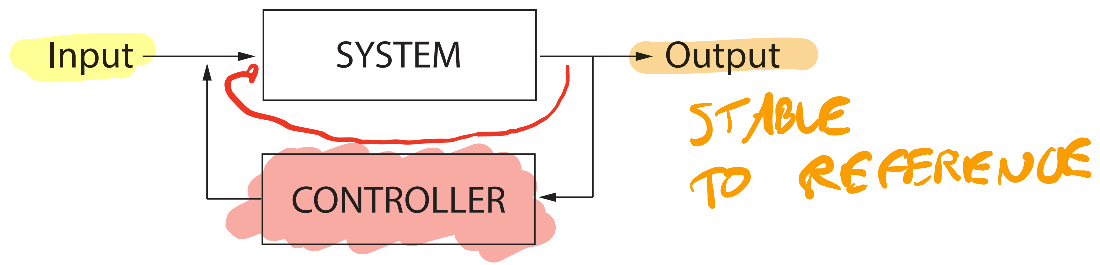


- SYST0017: Advanced topics in systems and control: goes further.
  (nonlinear systems, chaos, etc.)

Systems modeling in three courses

- SYST0002: Modeling and analysis of systems: **open loop**.
  "Observing and analyzing the environment"

- SYST0003: Linear control systems: **closed loop**.
  "Interacting with the environment"



- SYST0017: Advanced topics in systems and control: goes further.
  (nonlinear systems, chaos, etc.)

The concept of control

- Alice and Bob went out all night together.

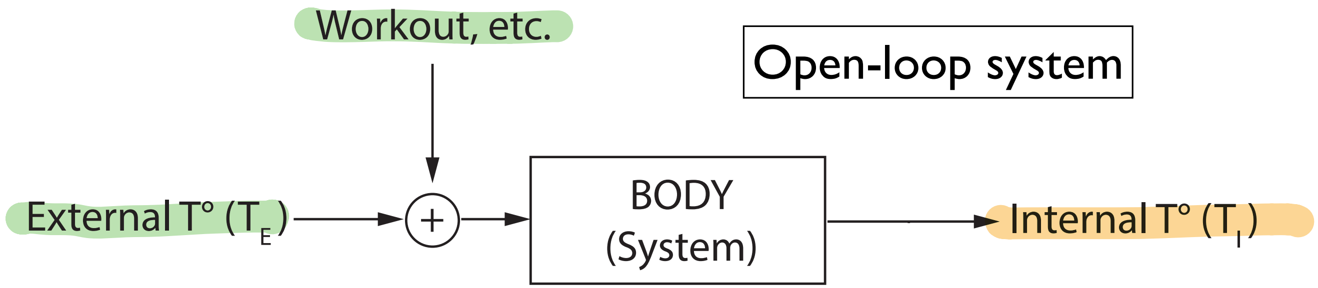
© Can Stock Photo - csp17115144

- The next day (it is 20°C outside):
  - **Alice** runs a 10km.
  - At the end of her run, her body T° = 39°C, so she
    - feels very **warm**.
      wears **thin clothes**.
      sweats a lot.
      wants to take a **cold shower**.
  - **Bob** wakes up really sick.
  - Like Alice, his body T° = 39°C, but he
    - feels very **cold**.
      lies underneath **several blankets**.
      shivers a lot.
      wants to take a **warm bath**.

OPEN-LOOP

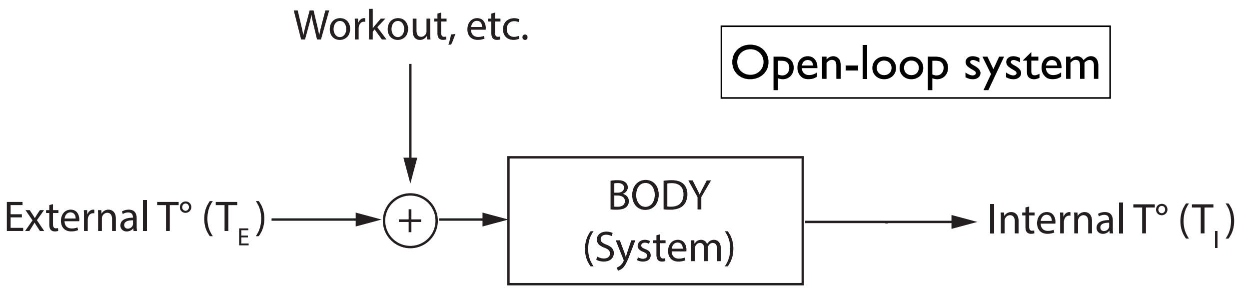
The control of body temperature

- Human body temperature is influenced by the environment and activity.



The control of body temperature

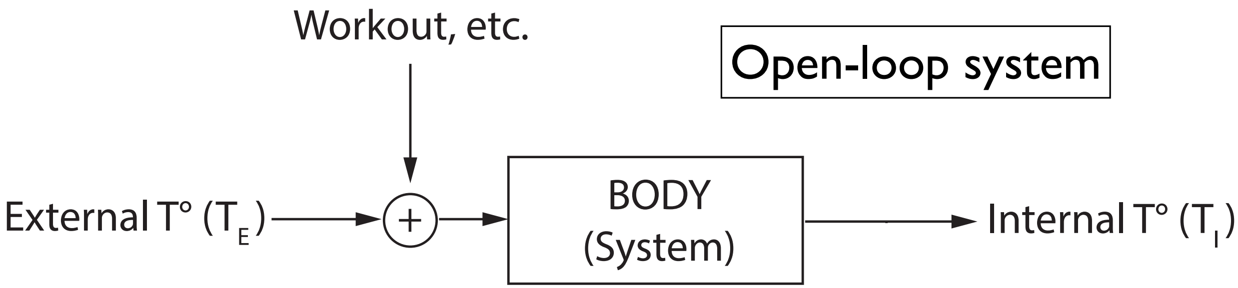
- Human body temperature is influenced by the environment and activity.



- How can we maintain a stable body temperature despite wide changes in the environment and/or changes in activity level?

The control of body temperature

- Human body temperature is influenced by the environment and activity.

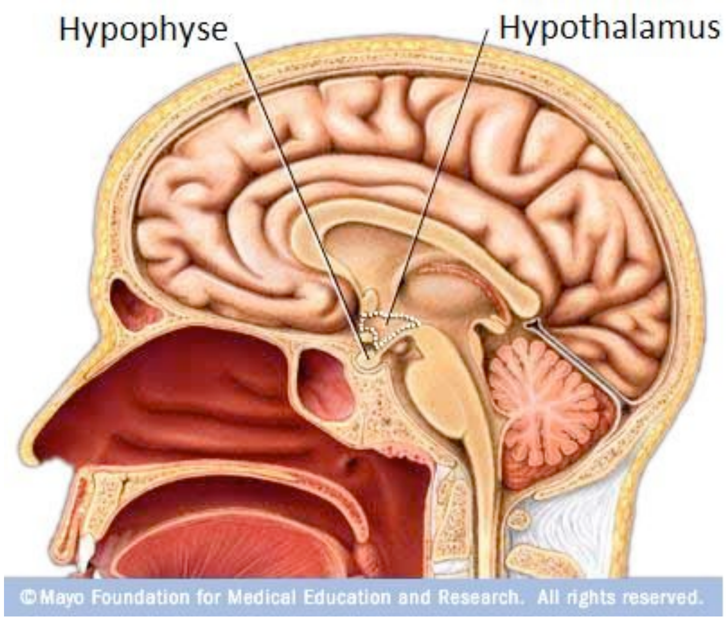


- How can we maintain a stable body temperature despite wide changes in the environment and/or changes in activity level?

- How does the body sense its own temperature? Why do Alice and Bob feel so different in the same conditions?

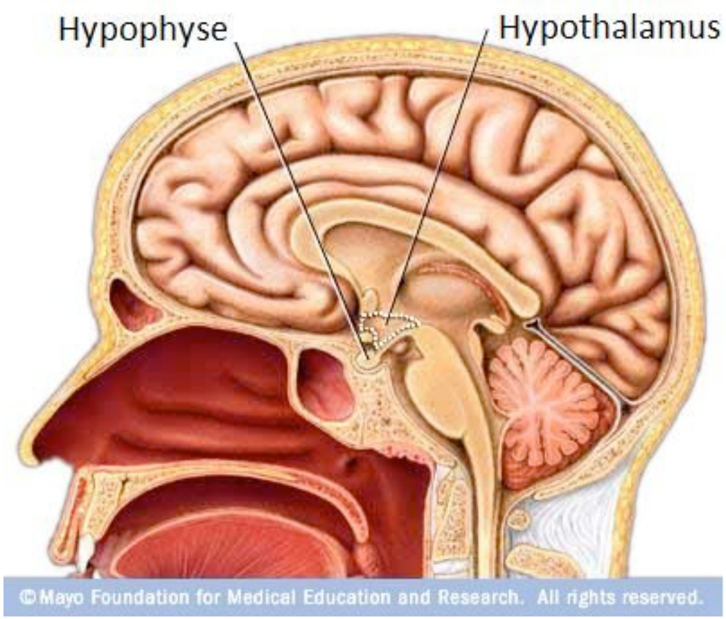
The hypothalamus: a temperature sensor.

- A part of the brain, called the **hypothalamus**, contains neurons whose activity is sensitive to changes in temperature.



The hypothalamus: a temperature sensor.

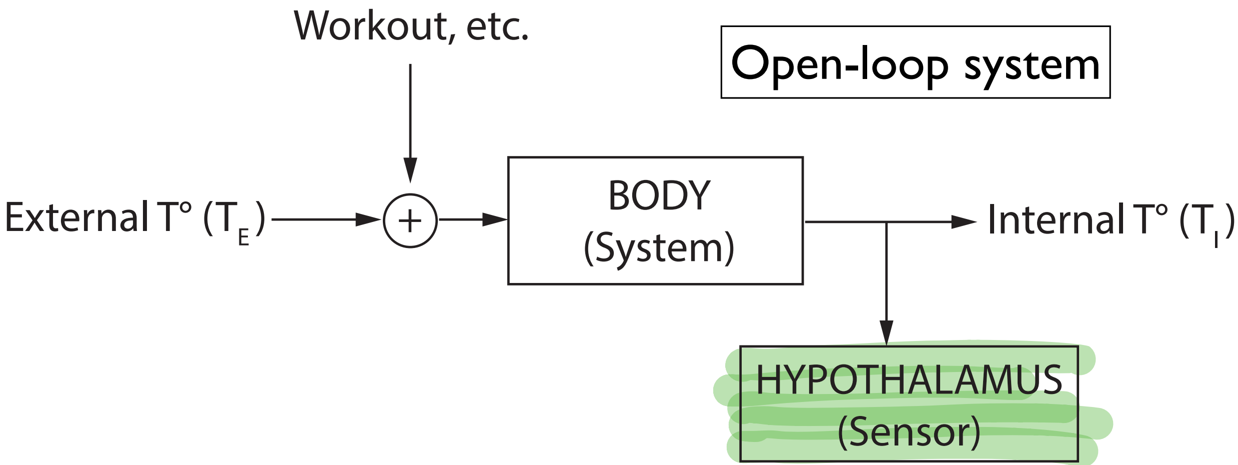
- A part of the brain, called the **hypothalamus**, contains neurons whose activity is sensitive to changes in temperature.



- More globally, these neurons represent temperature **sensors/controllers**. Sensors are critical components of a regulatory (control) system.

The control of body temperature

- Human body temperature is sensed by the hypothalamus.



- How can we maintain a stable body temperature despite wide changes in the environment and/or changes in activity level?

Cold or warm conditions induce a physiological response.

- If the hypothalamus senses that the temperature is whether too high or too low, it sends a message to induce **physiological responses**.

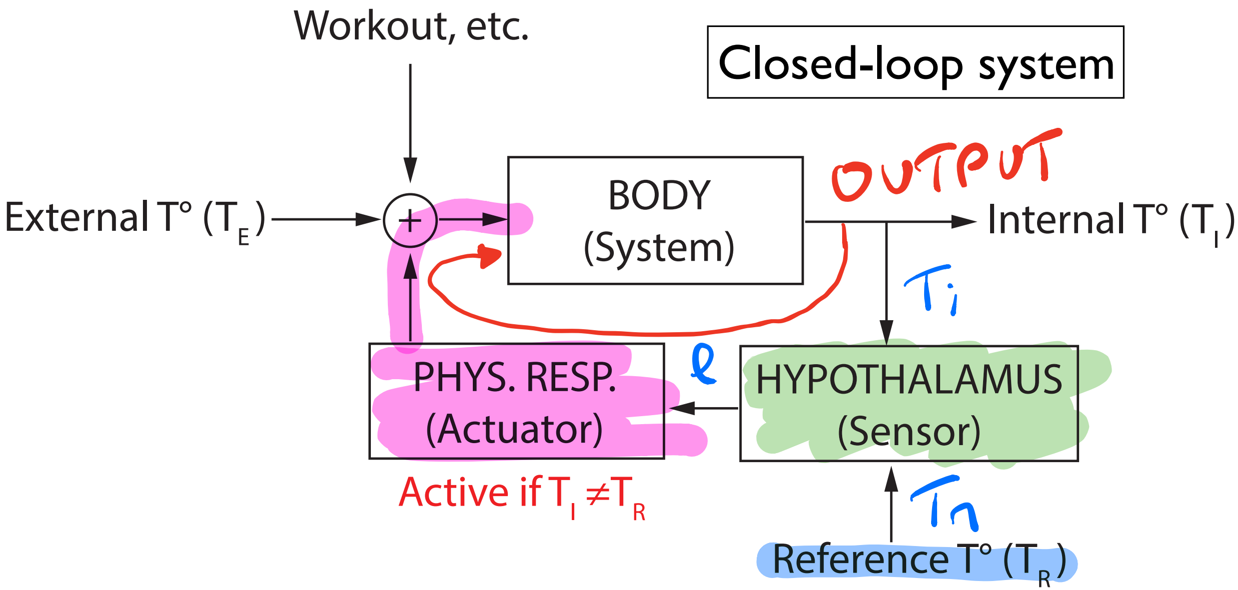
- **Cold**: vasoconstriction, shivering, curling up, warm clothing, heat source, etc.

- **Warm**: vasodilatation, sweating, lethargy, loose clothing, cooling, etc.

- These physiological responses are **actuators**. They actively increase or decrease the body temperature.

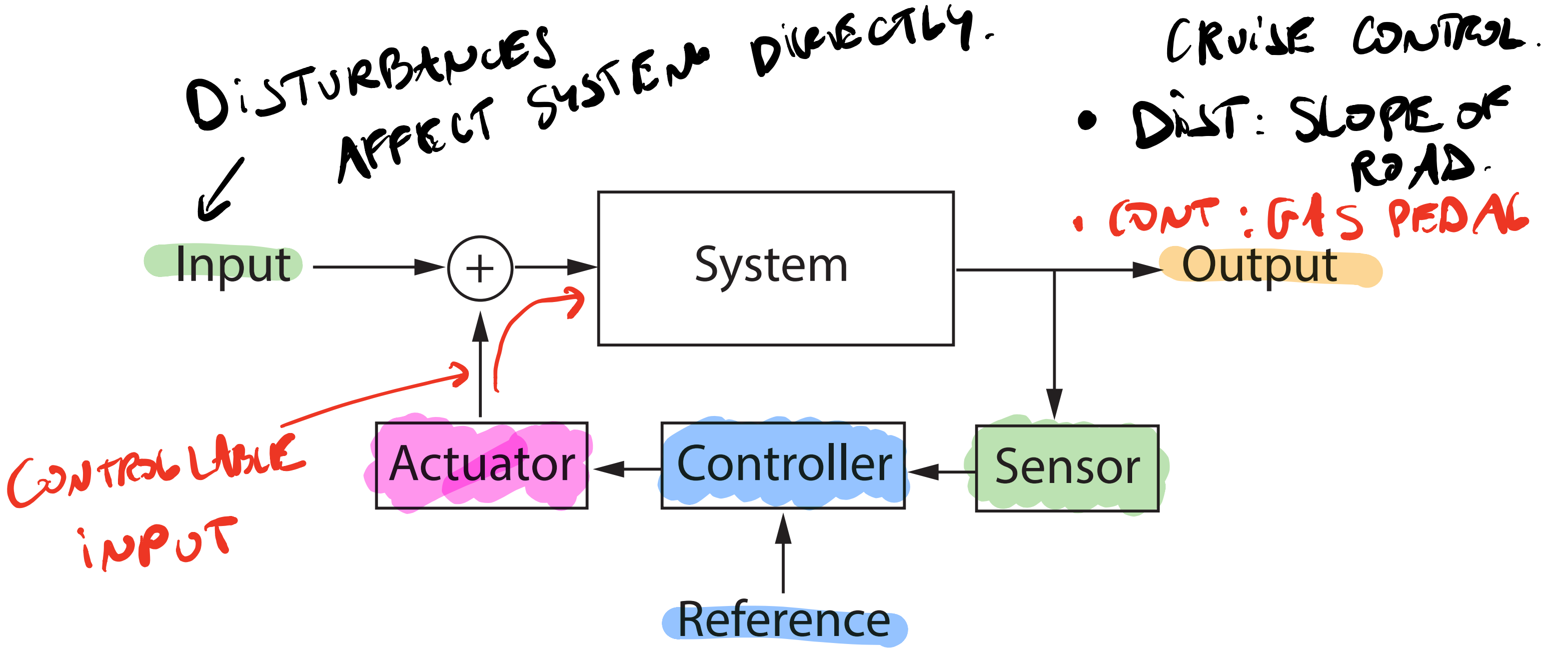
The control of body temperature

- Human body temperature is controlled by the hypothalamus/behavior.



- If the body temperature is different than the reference temperature, the hypothalamus induces physiological responses that tend to bring the temperature back to normal. It attempts to **correct the error** $e = T_I - T_R$.

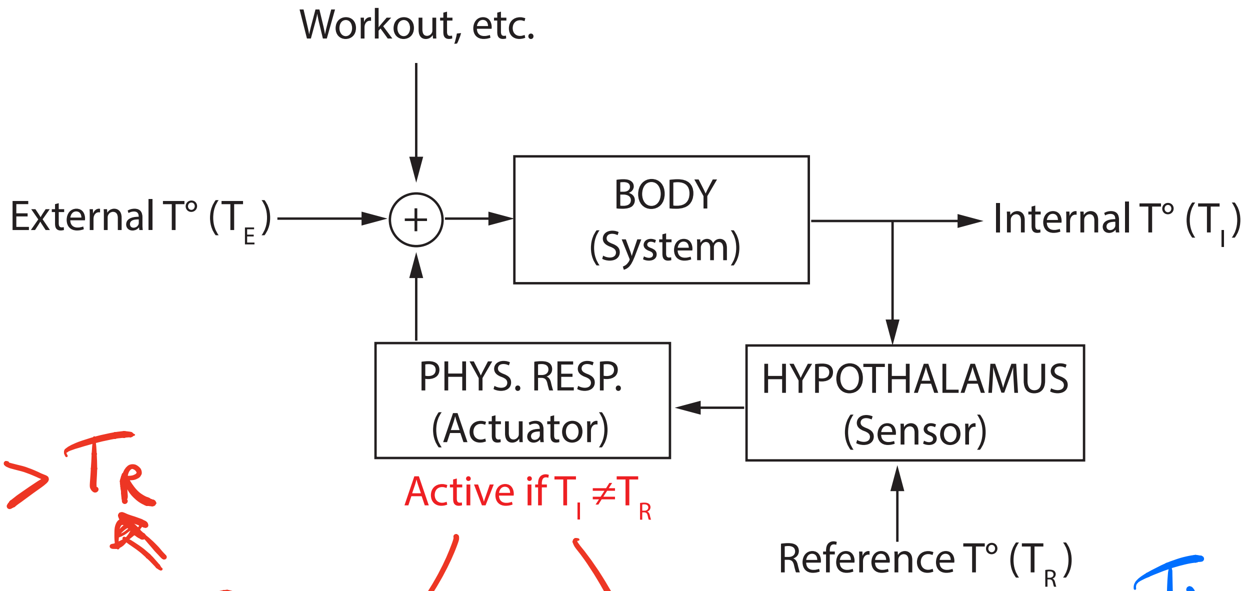
General structure of a **feedback** control system (closed-loop)



- ☐ The (negative) feedback loop acts against any deviation of the output from the reference value.

- ☐ First role of control: **robustness to uncertainty, disturbances, noise, etc.**

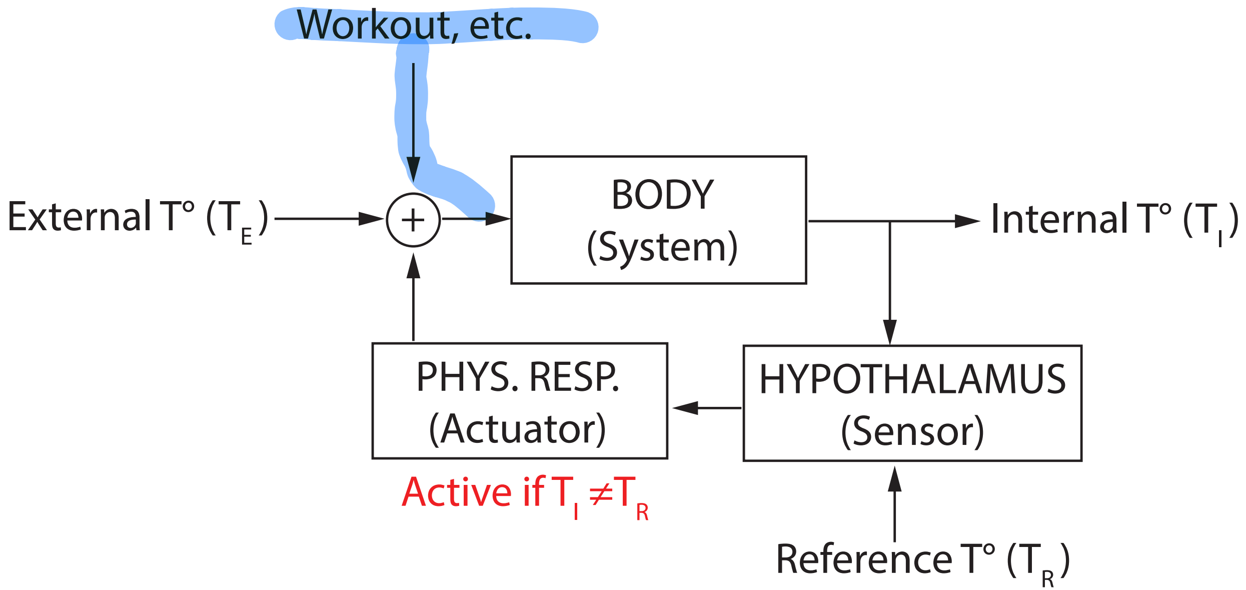
Back to Alice and Bob



- **Alice**:
  - feels very **warm**.
    wears **thin clothes**.
    sweats a lot.
    wants to take a **cold shower**.

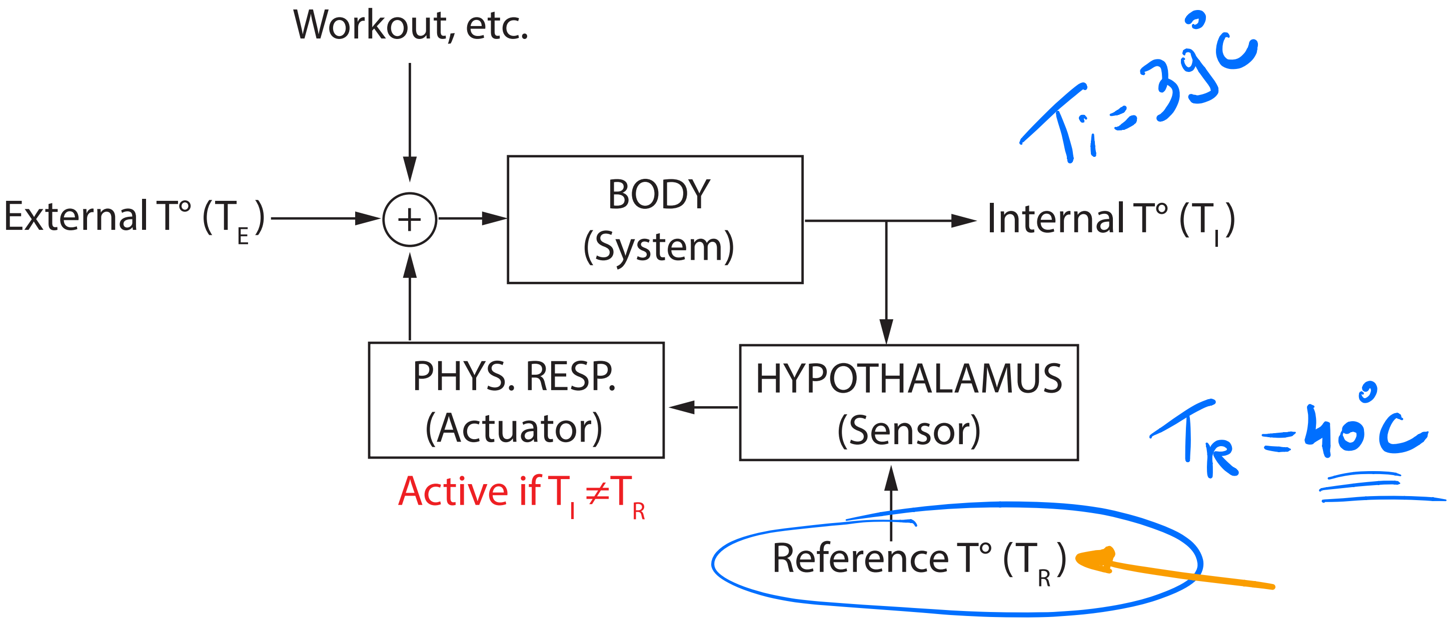
- **Bob**:
  - feels very **cold**.
    lies underneath **several blankets**.
    shivers a lot.
    wants to take a **warm bath**.

Alice: hyperthermia



- ☐ In the case of Alice, the workout has increased her body temperature.
- ☐ The thalamus sensed it, and induced a physiological response.
- ☐ The **controller works great**: it tends to reduce the increase in body T° induced by the workout. Why is it different for Bob?

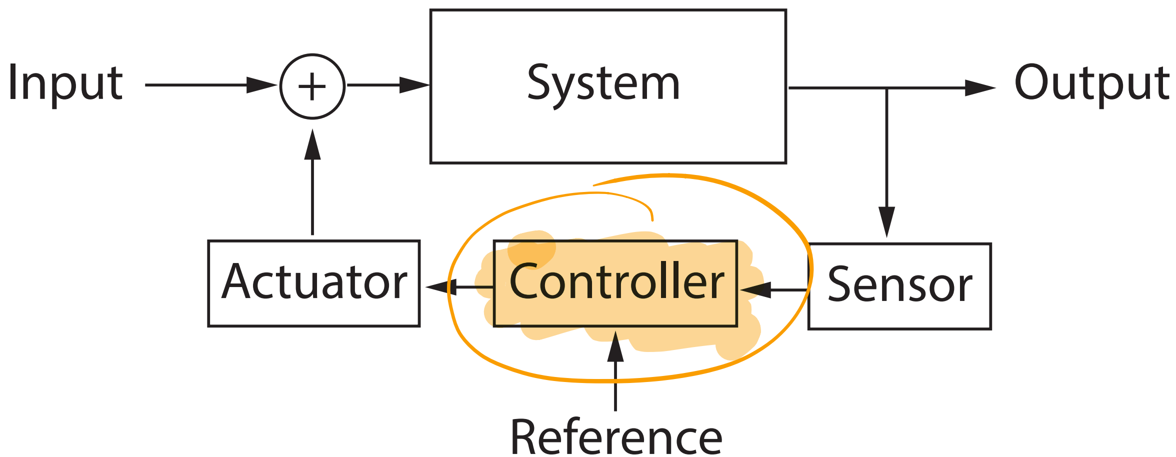
Bob: fever



- Bob has fever. Fever does not affect the body temperature directly, but **increases the reference temperature.**
- Because of the fever, the thalamus thinks Bob's body T° is too low.
- The high body temperature is **caused by the controller itself**!

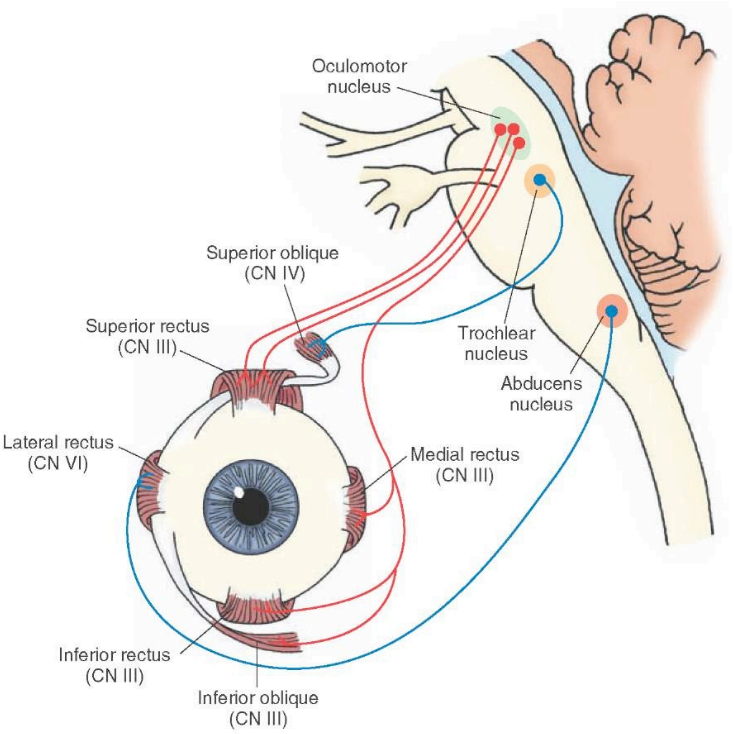
First role of feedback control

- If the controller senses that the output deviates from it reference value, it acts against this deviation: negative feedback.
- Control brings **robustness to uncertainty, disturbances, etc.** to the system.
- The controller can also have detrimental effects. It has to be **carefully designed**!



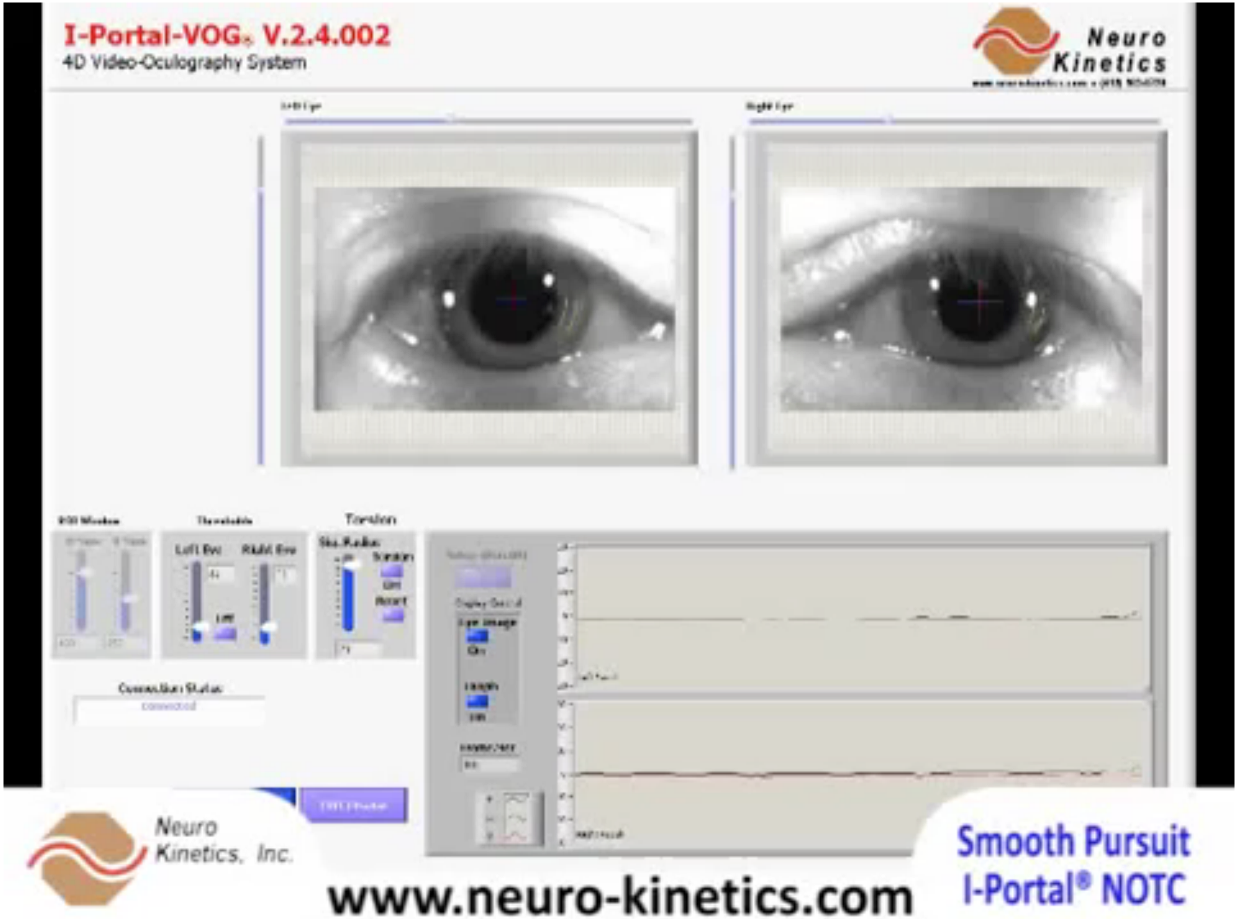
Example of a feedback system: eye movement.

- The eye movement is controlled to track a relevant part of the visual field.

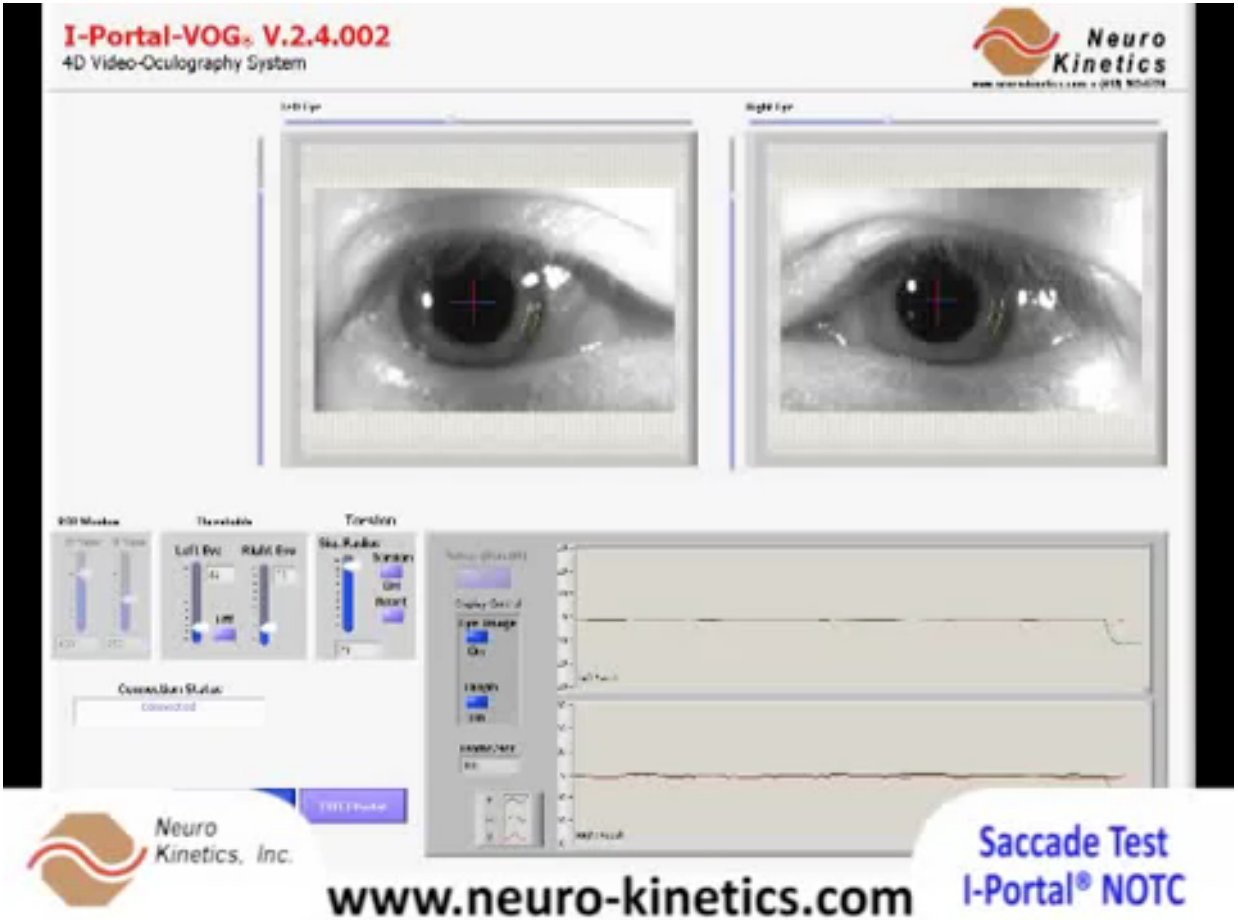


- To track a source, eyes use two types of movements: **smooth pursuits** and **saccades**.

Eye movement: smooth pursuit



Eye movement: saccades



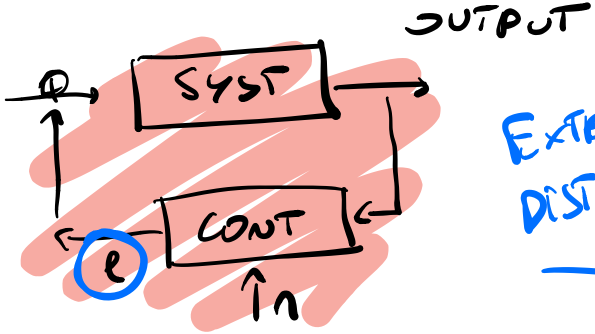
What is different between smooth pursuits and saccades?

- Both smooth pursuits and saccades can bring you to a specific place of the visual field. They both have similar **static performance**.

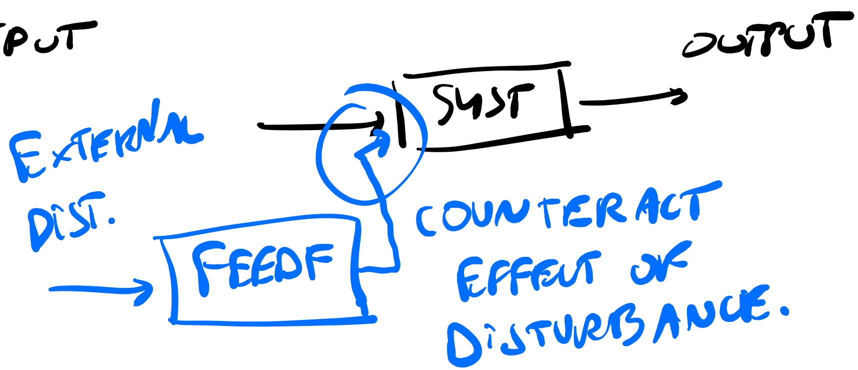
- However, smooth pursuits are slow and precise, whereas saccades are fast and coarse. They differ in their **dynamic performance**.

- Second role of control: **design of dynamics.**

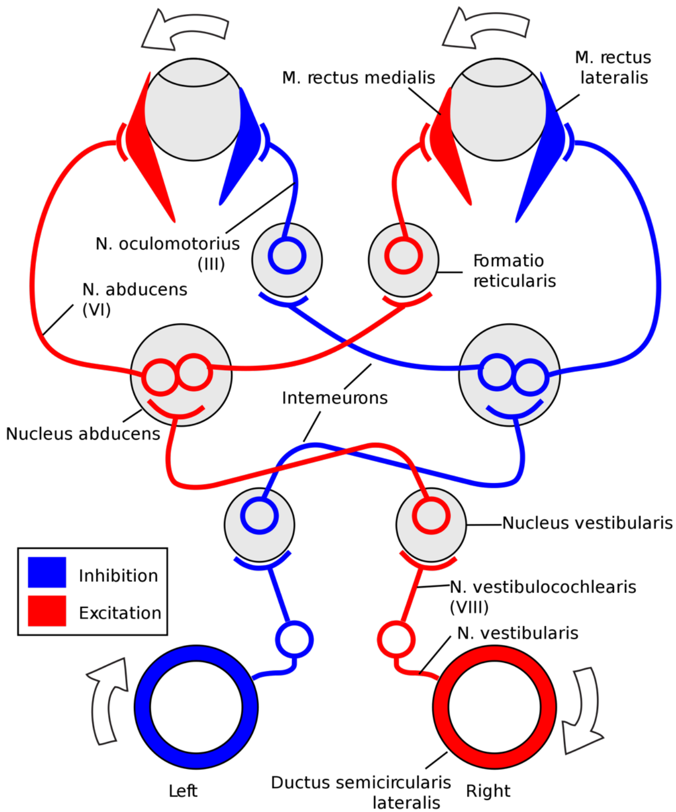
FEEDBACK. "REACTIVE"



FEEDFORWARD



Feedforward control: the vestibulo-ocular reflex



- The vestibular system senses if your head is moving.
- If it senses movement, it sends a signal to the eye muscles to generate an eye movement equal in amplitude but in the opposite direction.
- This reflex stabilizes the image on the retina during head movements (ex: reading in a train).
- This reflex can still occur in comatose patients.
- This type of control is called **feedforward** control.

Feedback control vs Feedforward control

- Feedforward control can be used when we can **measure the disturbance before it enters the system**.

- Feedforward control can be very efficient, but it requires measure/model of the disturbance outside of the system.


GUEST

Control can create instability: nystagmus

- In the video, the semicircular canals of the vestibular system are "still spinning". They therefore send a signal to the eye muscles to make the eyes turn.

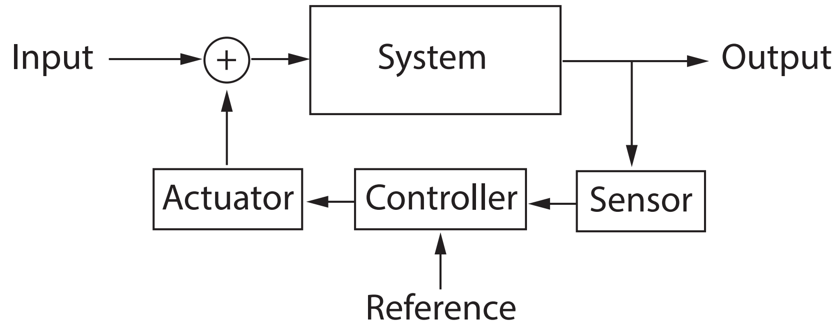
- On the other hand, the person tries to look at a fixed point in her visual space. The feedback system therefore attempts to counteract the movement generated by the feedforward system, but with a slight delay.

- This results in **instability** and oscillations (called **nystagmus** in this case).

The importance of a good design!

- Think about the control of body temperature. It has to be carefully designed:
  - If too slow: the person dies before the temperature is corrected.
  - If too fast: you can have big overshoots, oscillating between periods of coldness and periods of warmness.
- There is a trade-off between performance and robustness.
- This course will focus on how to design control systems.

The concept of control



- Properties:
  - Robustness to uncertainties, disturbances, noise, etc.
  - Design of dynamics: static performance vs dynamic performance.
  - Higher levels of automation.

- Drawbacks:
  - Can lead to instability. The controller has to be carefully designed!
  - Adds complexity to the system.

Higher level of automation

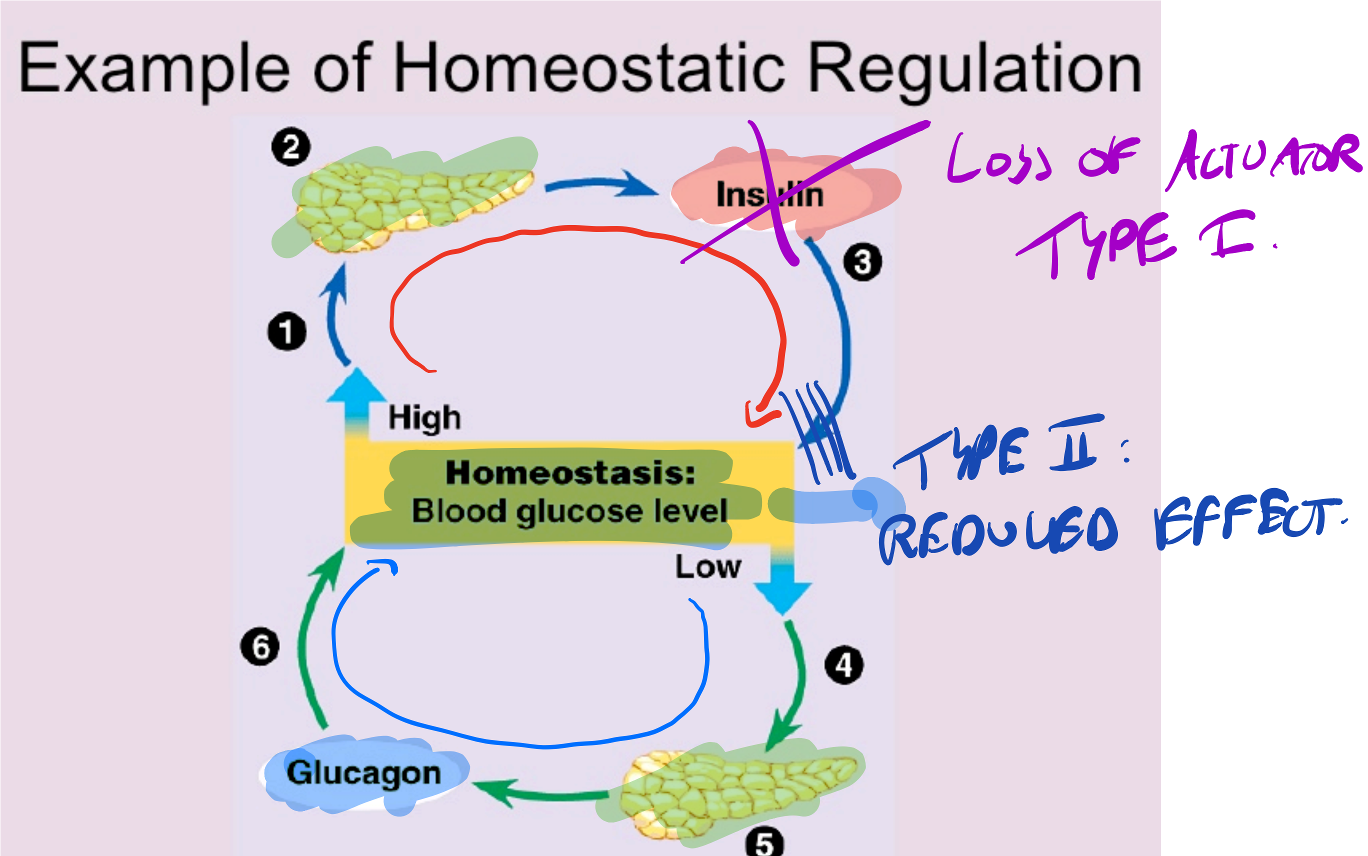
Examples of control systems

- Drones flying in formation

Alex Kushleyev, Daniel Mellinger, and Vijay Kumar
GRASP Lab, University of Pennsylvania

Examples of control systems

- Homeostasis and homeostatic regulation

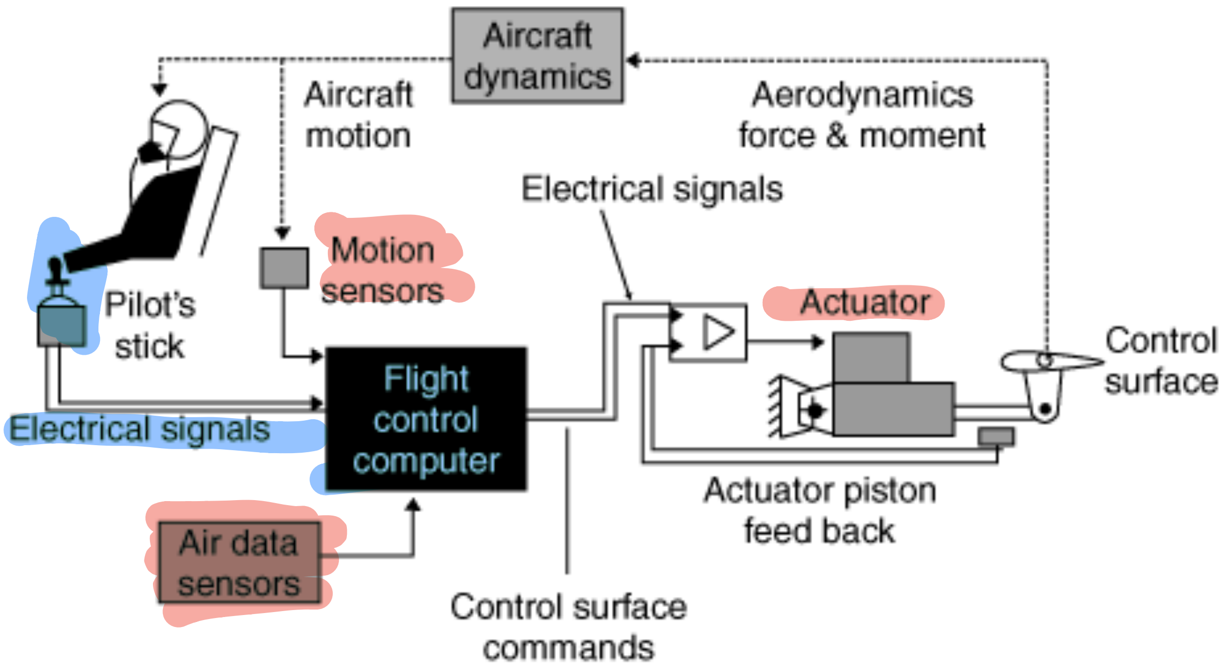


Examples of control systems

- SpaceX automatic landing.

Examples of control systems

- Fly-By-Wire

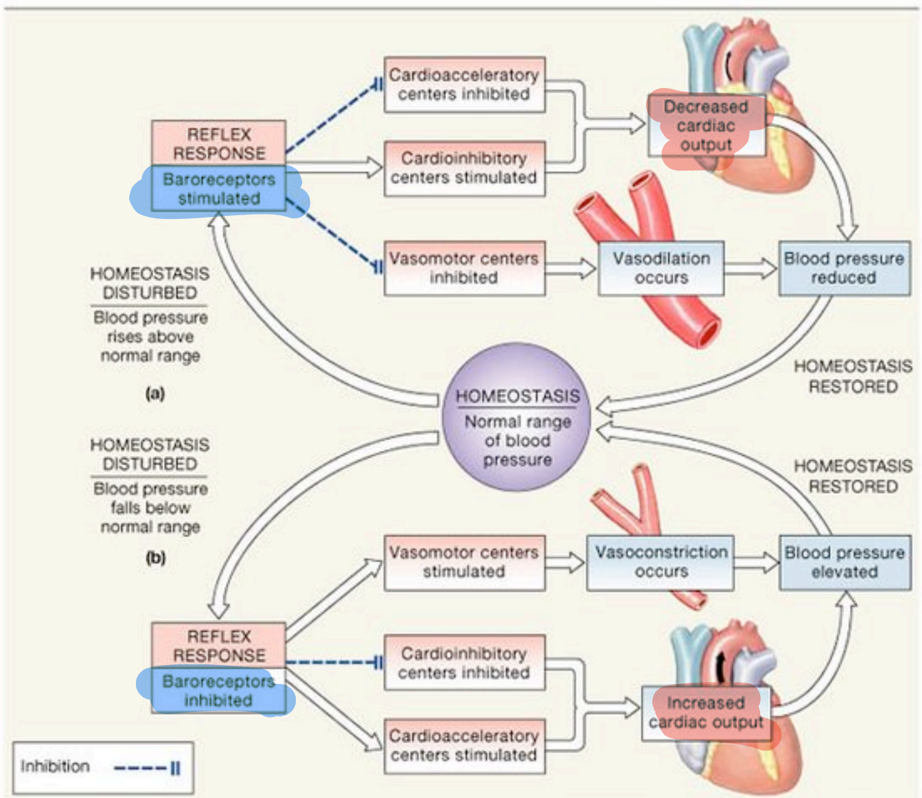


Examples of control systems

- Your phone, when you swipe between screens (sensitivity can be changed by changing the point where the reference changes).

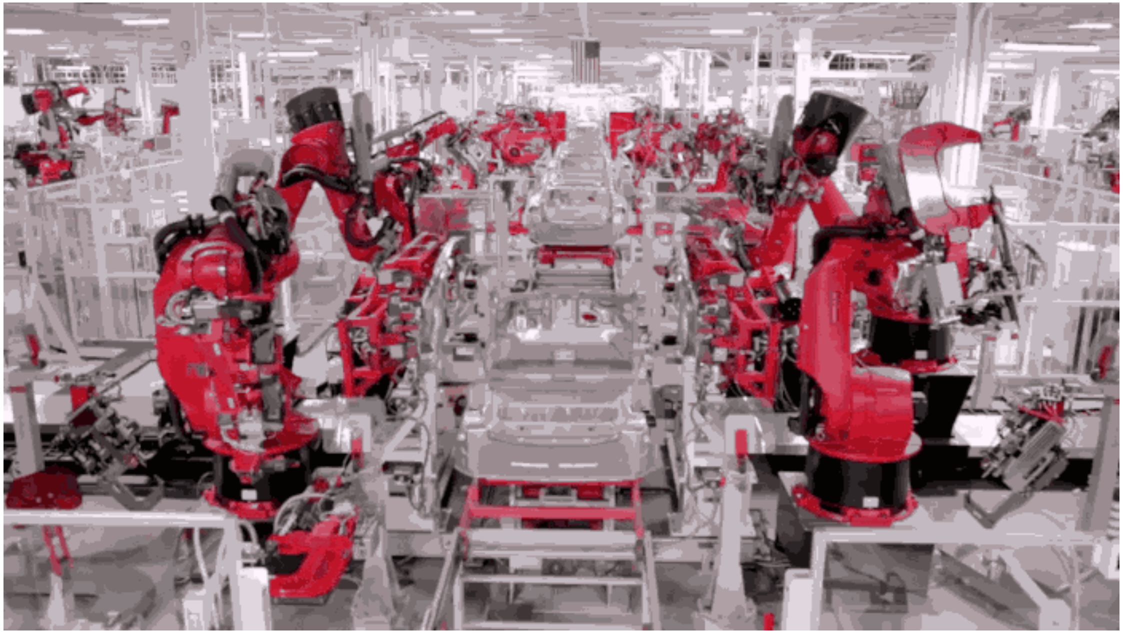
Examples of control systems

- Cardiovascular regulation



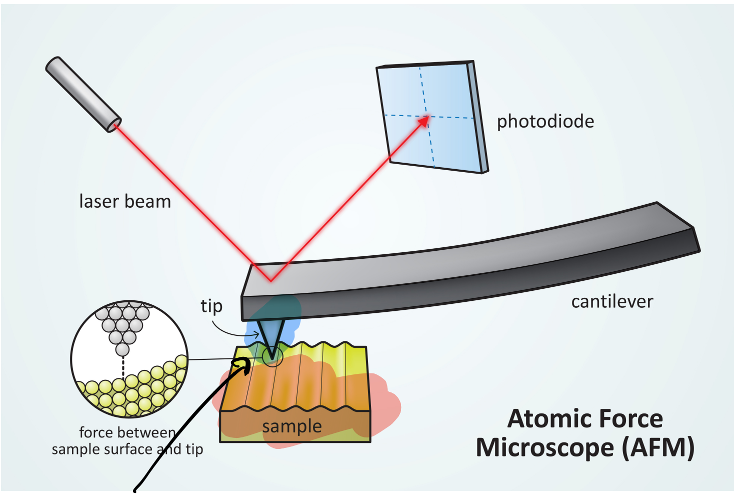
Examples of control systems

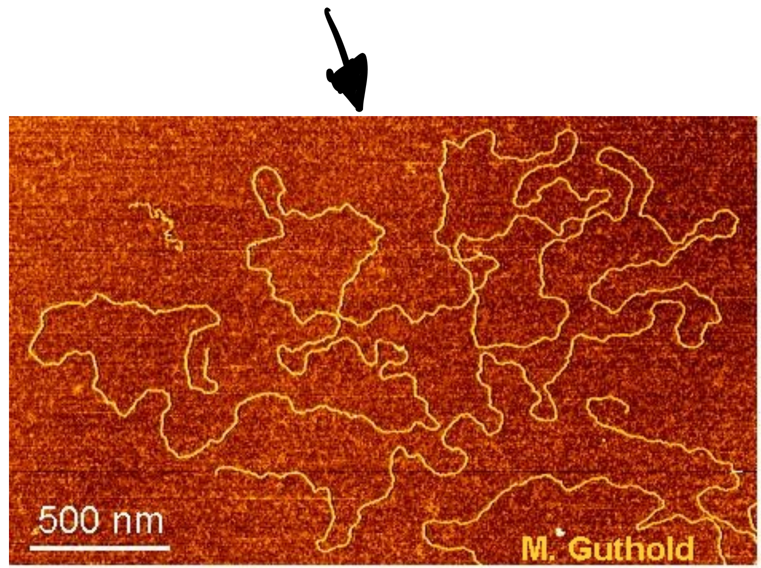
- Industrial robotics



Examples of control systems

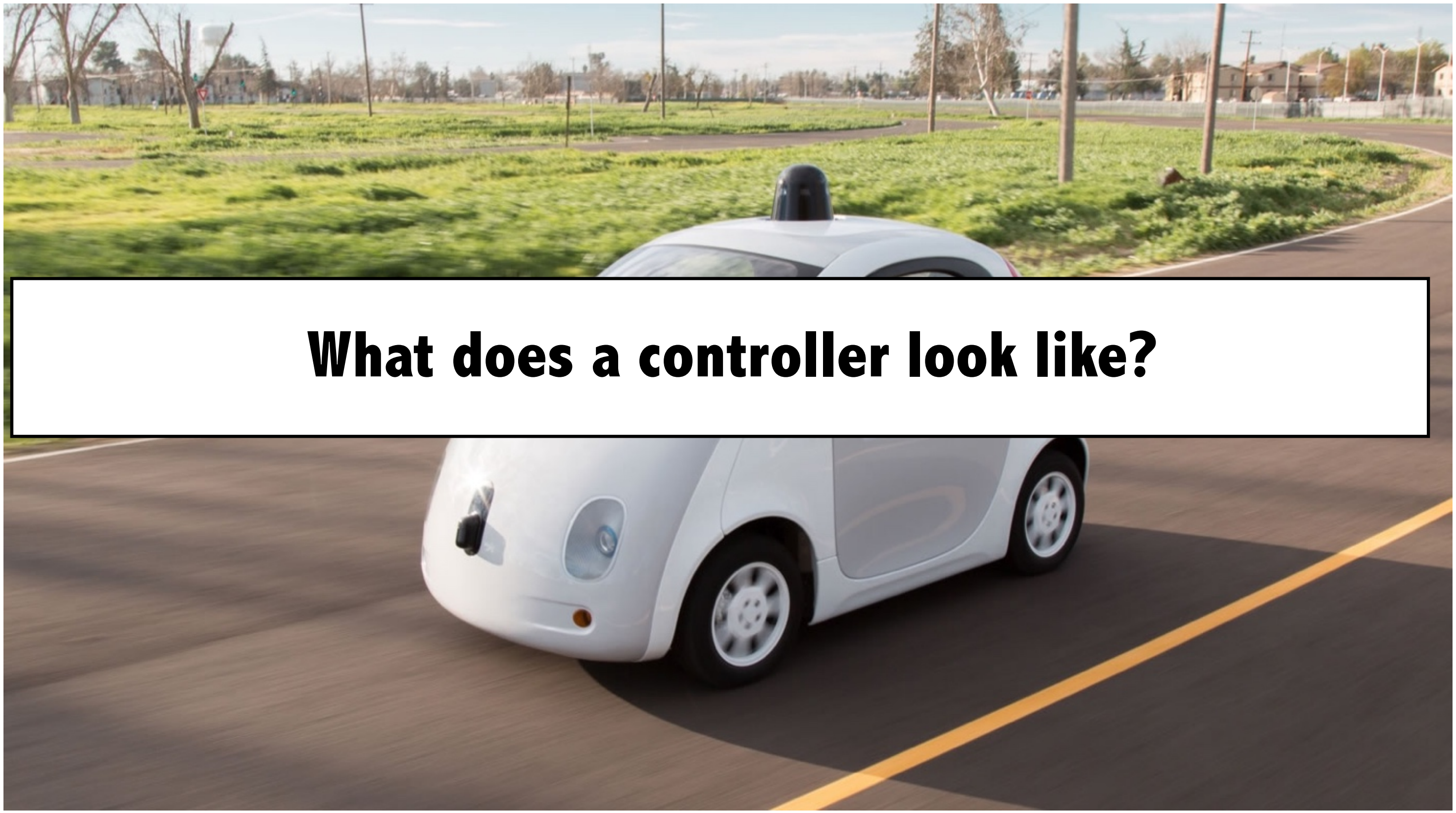
- Atomic force microscopy: measuring van der Waals forces of individual atoms





λ-DNA

Higher level of automation

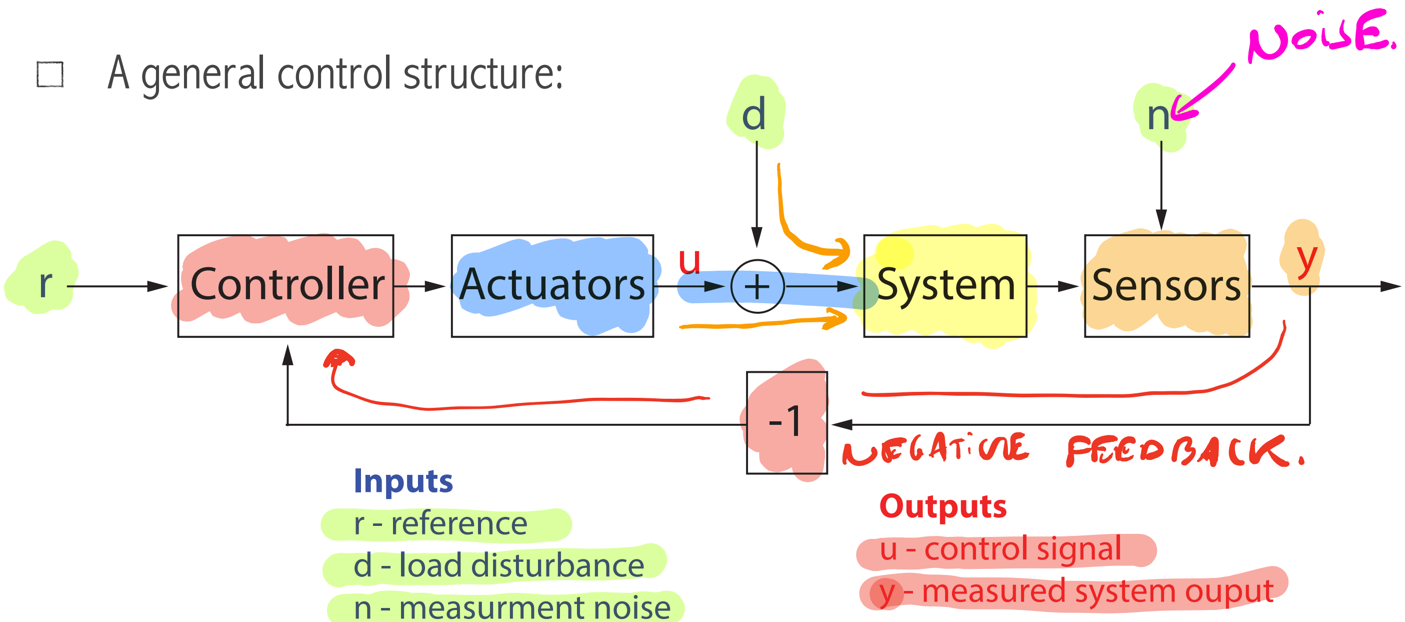


What does a controller look like?

Feedback control principle

- We want to design a controller such that the output of the closed-loop system **tracks a specific (and possibly varying) reference**.

- A general control structure:

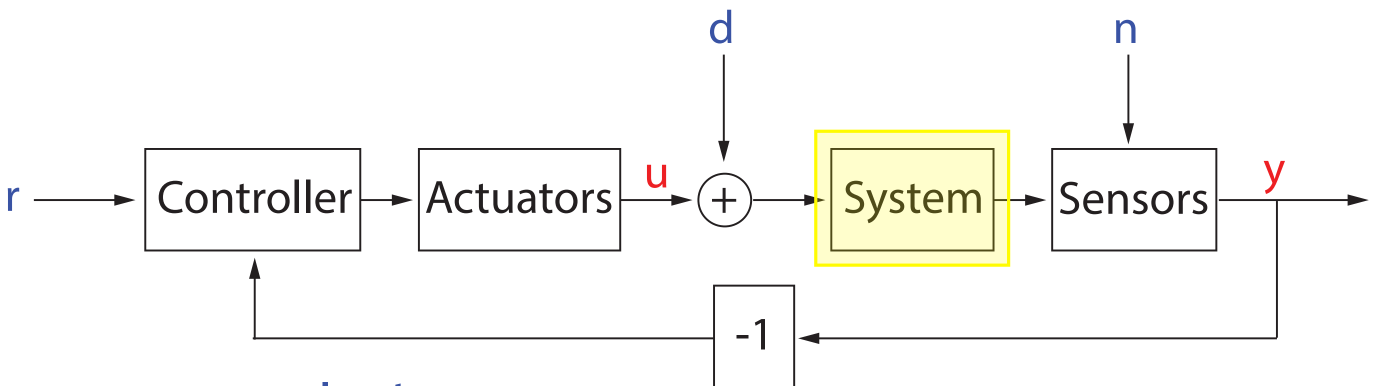


State-space representation of LTI systems

- First, we need a **state-space representation** of the **open-loop system**.

  ↳ MATHEMATICAL MODELING OF SYSTEM.

- This is where we start the design of the control (or closed-loop) system.



Inputs
r - reference
d - load disturbance
n - measurment noise

Outputs
u - control signal
y - measured system ouput

State-space representation of LTI systems

- A LTI system can be represented as follows

x : STATE VECTOR
y : OUTPUT u : INPUT.

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases} \qquad \begin{cases} \sigma^{-1}x = Ax + Bu \\ y = Cx + Du \end{cases}$$

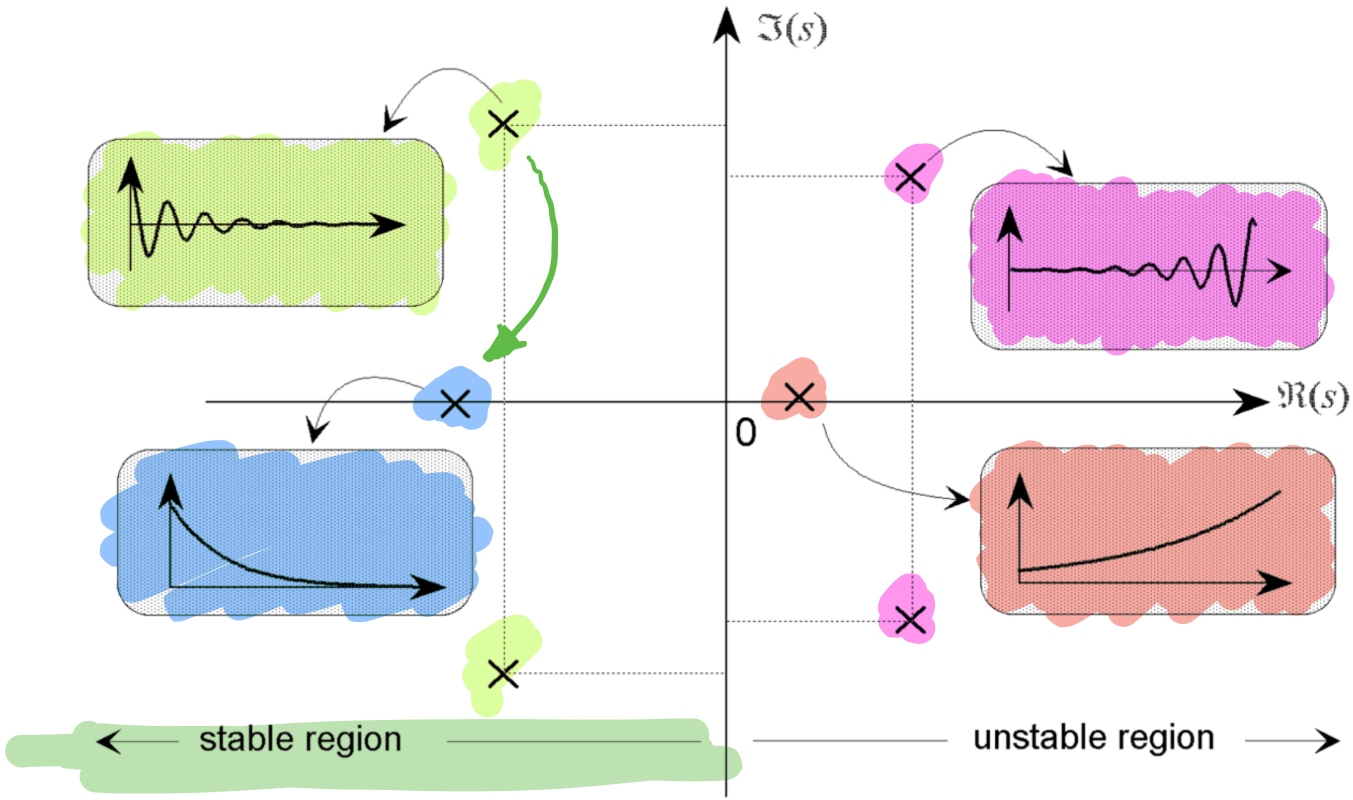
where x is the state vector whose dimension gives the dimension of the system.

- In a single-input/single-output (SISO) system, we have (ex: 4 states)

$$A = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{pmatrix} \quad B = \begin{pmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \end{pmatrix} \quad C = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot \end{pmatrix} \quad D = (\cdot)$$

State-space representation: dynamics and stability

- The dynamics and stability of a LTI system are determined by the **eigenvalues of the dynamics matrix A** (i.e. the poles of the transfer function).



State-space representation of LTI systems

- A LTI system can be represented as follows

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases} \qquad \begin{cases} \sigma^{-1}x = Ax + Bu \\ y = Cx + Du \end{cases}$$

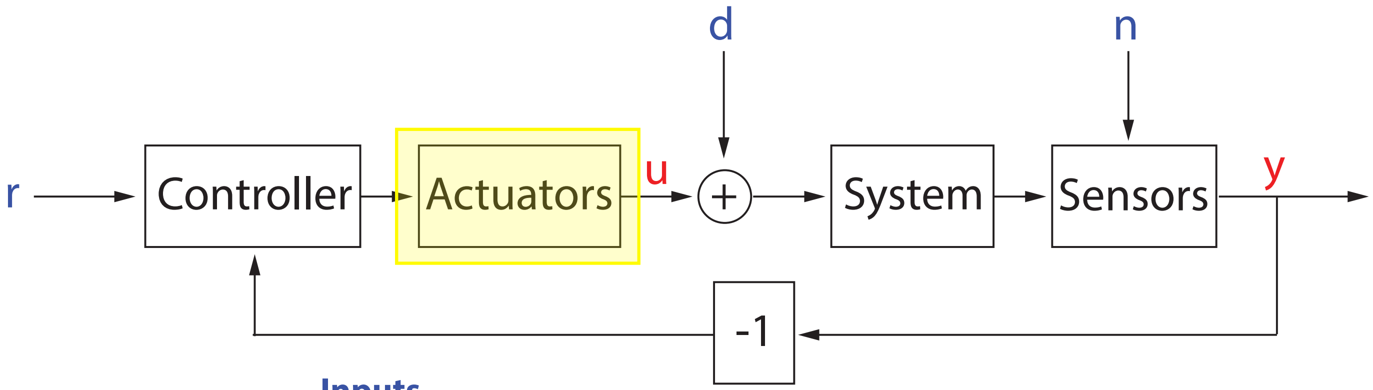
where x is the state vector whose dimension gives the dimension of the system.

- Regarding our control problem:
  - **A** is not tunable: **dynamics** of the open-loop system.
  - **B** is tunable: it is defined by the number and position of **actuators**.
  - **C** is tunable: it is defined by the number and position of **sensors**.

Feedback control principle

- Question #1: Can we control the states of the system acting only on its input (via the control signal)? Concept of **controllability**.

- i.e. how many **actuators** do I need, and what should they act on?



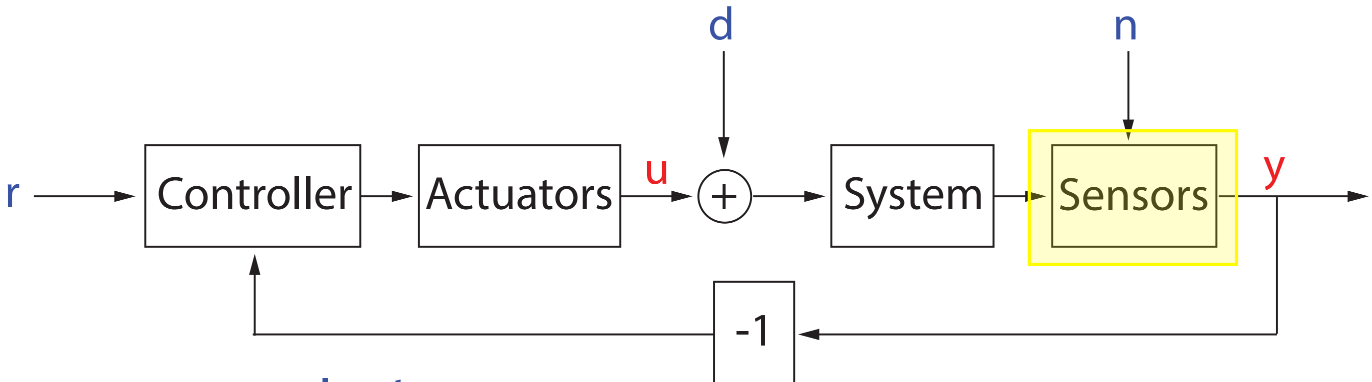
Inputs
r - reference
d - load disturbance
n - measurment noise

Outputs
u - control signal
y - measured system ouput

Feedback control principle

- Question #2: Can we infer the values of the states from the measured output? Concepts of **observability** and **state estimation**.

- i.e. how many **sensors** do I need, and what should they measure?



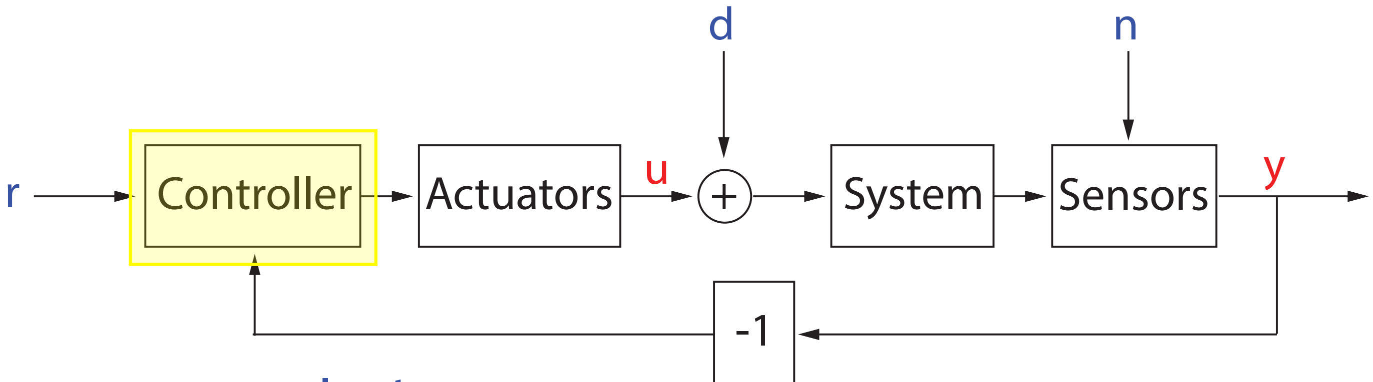
Inputs
r - reference
d - load disturbance
n - measurment noise

Outputs
u - control signal
y - measured system ouput

Feedback control principle

- Question #3: Provided that questions #1 and #2 are satisfied, how should I **design** my feedback controller? From **state-space** to **frequency domain**.

- i.e. what should be the **input-output properties of my controller** to reach the desired control action?



Inputs
r - reference
d - load disturbance
n - measurment noise

Outputs
u - control signal
y - measured system ouput

Key concepts

1. **Open-loop** vs **closed-loop**.
2. **Feedback** vs **feedforward**.
3. **Static performance** vs **dynamic performance**
4. **Stability**
5. **Performance/Robustness**