INFO2050 - Advanced computer programming

Exercise session 2: Recurrence and summation

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October 2014

Exercise 1

Find an analytic solution for:

(a) $\sum_{k=1}^{\infty}(2k+1)x^{2k}$ (with $|x|<1$);

(b) $\sum_{i=x}^{y}(2i+1)$;

(c) $\sum_{i=0}^{\infty}\sum_{j=1}^{n}\left(\frac{j}{j+2}\right)^{i}$.

(d) $\sum_{k=0}^{n}k^2 4^k$ (tips: use the perturbation method)

Exercise 2

Find an analytic solution for:

$$T(0)=2$$

$$T(n)=\frac{1}{n}\sum_{p=0}^{n-1}T(n-1-p)T(p) \quad \forall n>0$$

Exercise 3

Find an analytic solution for:

$$T(1)=1$$

$$nT(n)=(n-2)T(n-1)+2 \quad \forall n>1$$

Exercise 4

Find an analytic solution for:

$$T(1) = 13$$
$$T(n) = 2T(n/8) + 21n \quad \forall n > 1$$

Exercise 5

Consider the following recurrence (where $n > 1$ is a power of 3):

$$T(1) = 0$$
$$T(n) = 6T(n/3) + 2n$$

(a) Without solving the recurrence, show that $T(n) \in O(n^2)$ (tips: refer to the *master theorem*).

(b) Find an analytic solution.

Exercise 6

For each of the following pseudo-codes, determine what is doing the algorithm and its asymptotic complexity (be precise with the notations).

```
CODE1(n)
1  if n ≤ 1
2        return n
3  else
4        return CODE1(n - 1) + CODE1(n - 1)
```

```
CODE2(n)
1  if n == 0
2        return ""
3  else
4        tmp = CODE2(n/2)
5        if n%2 == 0
6              return tmp + tmp
7        else
8              return tmp + tmp + "x"
```

```
CODE3(A, k)
1  for i = 1 to A.length
2        if A[i] == k
3              return i
4  return −1
```

Bonus

Bonus 1

In a machine learning applicaiton, we would like to extract contiguous substrings from a reference string A of length n. How many substrings of length k, at most k and between lengths k_1 and k_2 are there ? How many in total ?

Bonus 2

The first Euler project (https://projecteuler.net) does not require a computer:

> *"If we list all the natural numbers below 10 that are multiples of 3 or 5, we get 3, 5, 6 and 9. The sum of these multiples is 23. Find the sum of all the multiples of 3 or 5 below 1000."*