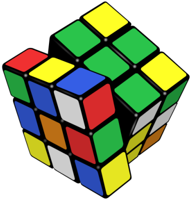
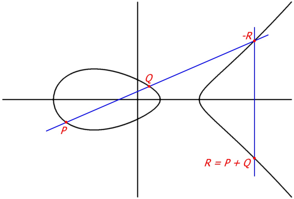
From Rubik's to cryptography

A tour of computational challenges in the field

Christophe Petit





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- 1567 : marries James Hepburn
- 1567 : forced to abdicate, she **flies to England**

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- Conspiracy suspected but Queen Elisabeth needs proofs

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- Principal secretary **Walsingham**, also chief of intelligence services, puts **Thomas Phelippes** on duty to **break Mary's code**
- Mary's life now relies on the strength of her cipher. . .

Outline

Elliptic curve cryptography

Hash functions and the Rubik's cube

Side-channel attacks

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Cryptography

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- Securing communication in the presence of *adversaries*
  - Confidentiality
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- Building blocks : encryption, MACs, signature, ...
- Many applications today : ATM cards, computer passwords, electronic commerce, electronic voting,...

Cryptography Wall of Fame

- Julius Caesar
- Abu al-Kindi
- Blaise de Vigenère
- Charles Babagge
- Auguste Kerckhoffs (ULG !)
- Claude Shannon
- Alan Turing
- Whitfield Diffie and Martin Hellman
- Ronald Rivest, Adi Shamir and Leonard Adleman
- Neal Koblitz and Victor Miller
- ...

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- Good news : some computational problems seem hard

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  find k such that $h = g^k \bmod p$
- **Elliptic curve discrete logarithm** (ECDLP)
  Similar as DLP but multiplicative group of a finite field replaced by group of points of an elliptic curve (see below)

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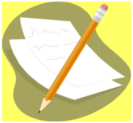
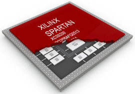
- Many variants of previous problems
- ...

Strength of the assumptions

- Some are stronger than others
- Depends on the **size** of parameters
- Evaluated based on
  - Best algorithms
  - Computing power
  - Fame of the problem
- See `www.keylength.com`





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- Recovering m from $g^m \bmod p$ (or b from $g^b \bmod p$) is the **discrete logarithm problem**

Symmetric key vs. Public key

- In **symmetric key** cryptography,
  single secret key shared between sender and receiver
- In **public key** cryptography, one key is public, but only one person knows corresponding secret key
  - Everybody can encrypt, only one can decrypt
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- **Key management** harder for symmetric keys
- Symmetric key algorithms often more **efficient**
- Public key algorithms rely on "simpler and nicer" **complexity assumptions**

General advices to Mary Stuart

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  - Use long term public keys to derive session secret keys
- Beware of authentication issues !
  - Textbook Diffie-Hellman can be broken with a simple man-in-the-middle attack
  - Use certificates to authenticate public keys

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- 1985 : Koblitz and Miller independently proposed to use the **group of points of an elliptic curve** instead

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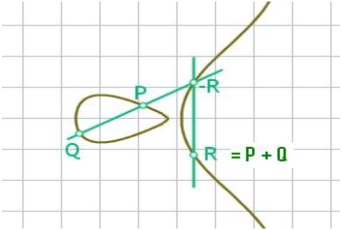

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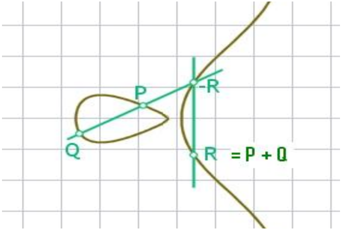

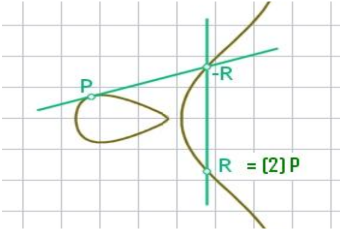

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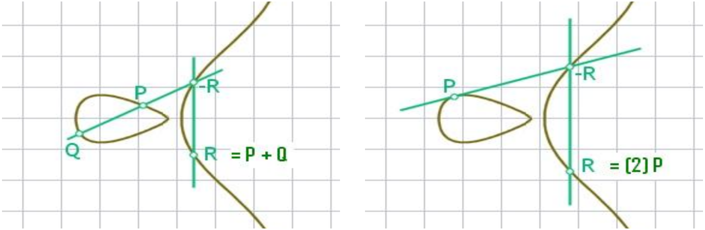

- **Elliptic curve discrete logarithm problem**
  Given P and $Q = [k]P$, find k

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- 2009 : NSA advocates use of ECC

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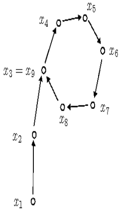
- When a **collision** $P_m = P_n$ is found, Deduce $Q = \left[\frac{a_m - a_n}{b_m - b_n}\right] P$



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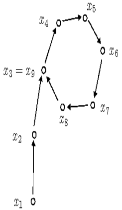

- Time complexity $\approx |G|^{1/2} \Rightarrow$ today we need $|G| > 2^{160}$

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- Can be adapted for factoring

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- For $B \approx \exp((\log p)^{1/2})$, **subexponential complexity**

Index calculus in practice

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- Main costs include power and building costs. . .

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- Avoid binary curves for at least five years
- Beware that algorithm improvements are more likely to come for ECDLP than DLP or factoring

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Hash functions and the Rubik's cube

Side-channel attacks

Cryptographic hash functions

- "Compressing" functions

$$H : \{0,1\}^* \to \{0,1\}^n$$

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- Often used as "pseudo-random functions"

Applications

- Message authentication codes
- Digital signatures
- Password storage
- Pseudorandom number generation
- Entropy extraction
- Key derivation techniques
- ...
- ...

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- All of them have “block cipher-like strucure”

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- **Goal** : relate main security properties of a hash function to "simple" hard problems from **group/graph theory**

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- Write $m = m_1 m_2 ... m_N$ with $m_i \in \{0, ..., k-1\}$
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- Efficiency can be good, depending on G and S
- Parallelism : $H(m||m') = H(m)H(m')$

Cayley graph perspective

- Hash computation $\sim$ *walk* in the *Cayley graph*

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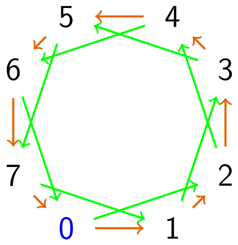
- Example : $G = (\mathbb{Z}/8\mathbb{Z}, +)$, $S = \{1, 2\}$



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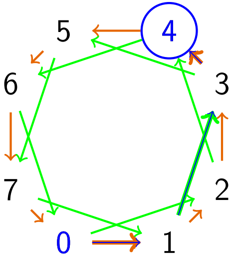

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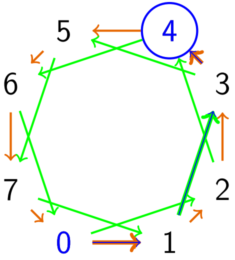

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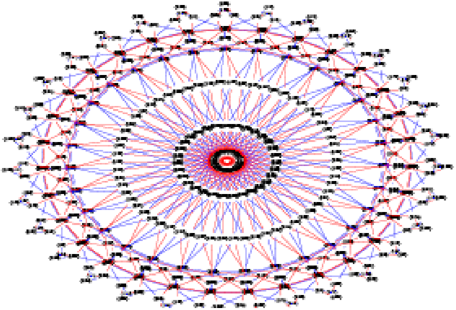
- Preimage algorithm $\sim$ path-finding algorithm

Example : Tillich-Zémor [TZ94]

$$G = SL(2, \mathbb{F}_{2^n}), \qquad S = \left\{ A_0 = \begin{pmatrix} X & 1 \\ 1 & 0 \end{pmatrix}, A_1 = \begin{pmatrix} X & X+1 \\ 1 & 1 \end{pmatrix} \right\}$$



A hard (?) problem

- **Factorization problem in finite groups** :
  Given G, $g \in G$ and $S = \{s_0, ..., s_{k-1}\} \subset G$,
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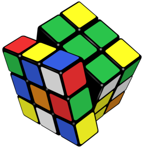
- Has this problem been sufficiently studied ?

Popular example : the Rubik's cube



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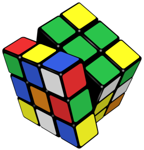

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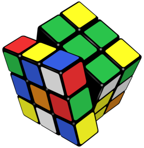

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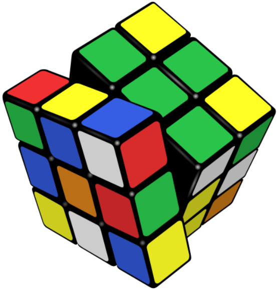
- Generated by the faces' rotations
- Neutral element $\sim$ Rubik's cube when solved
- Solution $=$ combination of the elementary permutations leading to the neutral element

Is Rubik hard enough ?



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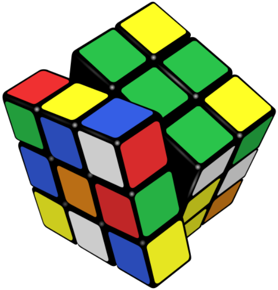

Not really, but generalizations might be

Related problems

- **Babai's conjecture** [BS92]
  - *There is a constant c such that, for any non-Abelian finite simple group G, for all generator sets S, the diameter of the Cayley graph arising from G and S is smaller than $(\log|G|)^c$.*
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- **Expander graphs**
  - Cayley graphs tend to be good expanders
  - Expanders have a lot of applications [HLW06]
  - Traveling in those graphs will be useful, too

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- Other **particular parameters** broken
- **General parameters** : work in progress

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- Use MACs or signatures to authenticate the messages
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- Too risky to use hash functions from Cayley graphs
- Working on generalizations of the Rubik's cube will be a funny and useful way to spend your time in prison
  - Expander graphs and their applications
  - Babai's conjecture
  - Cryptographic applications

Outline

Elliptic curve cryptography

Hash functions and the Rubik's cube

Side-channel attacks

An example of side-channel attack

- Choose a random bit $b \in \{0, 1\}$. Keep it perfectly secret.

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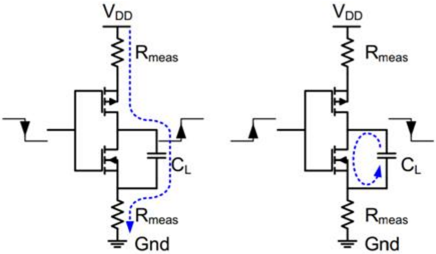
- From z only, I know nothing about b
- From **computing time**, I can guess b with a good probability.

CMOS inverter dynamic consumption



Charge vs. discharge of a CMOS inverter.

Figure credit : FX Standaert

$$P = C_L V_{DD}^2 P_{0 \to 1} f$$

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- In most crypto algorithms, recovering the private key from the messages would require solving a very hard problem
- **Side-channel attacks** : use computing side information
  - Timing, computing power, electromagnetic variations, keyboard noise,. . .
- **Fault attacks** : induce faults during computation, deduce relevant information from the result
  - Alter memory
  - Skip some instructions

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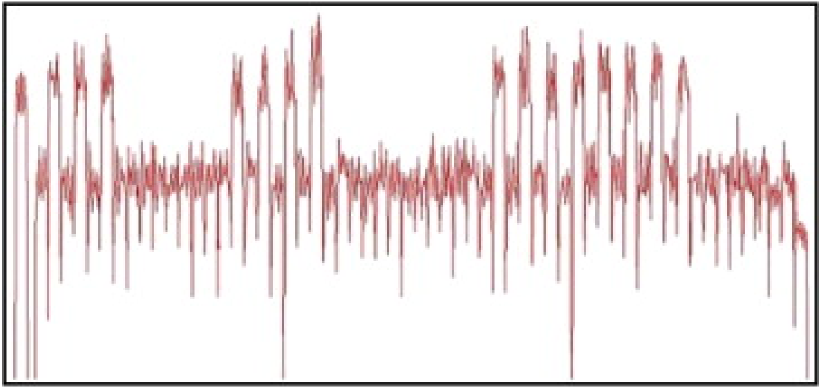
- What is the power consumption ?

A power attack against SM



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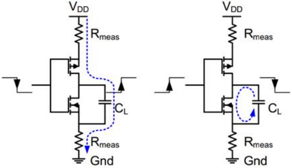

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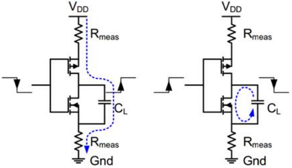

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- **Correlation attack**
  - Make a guess on a key byte
  - Deduce Hamming weight (variations) of the registers
  - Correlate with the power trace(s)

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- **Brute-force** to eliminate last wrong key candidates

Countermeasures

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  - Physical and chemical shields
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- Algorithmic countermeasures
  - Dummy operations
  - Noise addition
  - Masking
  - Shuffling
  - ...

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- If possible, build them from standard algorithms

A general advice to Mary Stuart

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- Beware that even a secure algorithm can become unsecure if badly implemented
- Include appropriate side-channel counter-measures in your favorite crypto computing machine

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“Cipher of Death”

- Mary Stuart **didn’t use good crypto**
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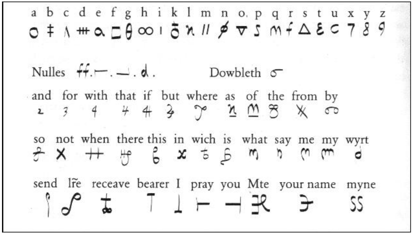

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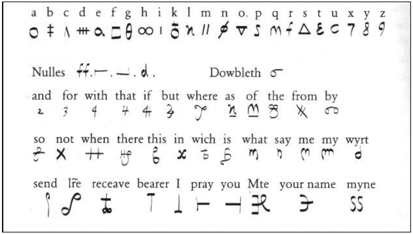

- Walsingham sent her a **fake message** asking confirmation of her commitment ; she answered
- Mary sentenced to death and executed on Feb 8th, 1587

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- Challenges for the bad (?) guy
  - New algorithms for hard problems (ECDLP,. . . ...)
  - Perform huge cryptanalysis tasks
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Credits

- The first chapter of Simon Singh's *Code Book* clearly inspired the introduction of this talk.

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- Everybody can encrypt, but private key needed to decrypt
- Computing (p, q) from the public key is the **integer factorization problem**

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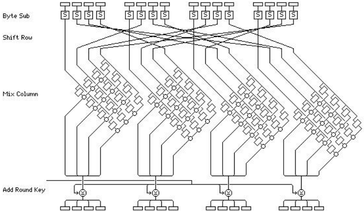
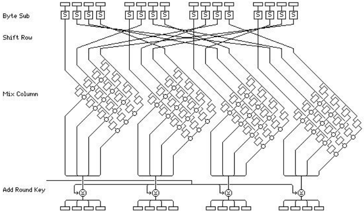
- Key size of 128, 192, or 256 bits
- Block size 128 bits
- 10, 12, or 14 rounds



Advanced Encryption Standard (AES)

- Algorithm Rijndael (Vincent Rijmen and Joan Daemen)
- Selected in 2001 after public competition
- Replaced previous standards DES and 3-DES

- Key size of 128, 192, or 256 bits
- Block size 128 bits
- 10, 12, or 14 rounds



- **Assumption** : AES good pseudo-random permutation

Example : a naive index calculus for $\mathbb{F}_p^*$

- DLP : given $g, h \in \mathbb{F}_p^*$, find k such that $h = g^k$
- Factor basis made of **small "primes"**

$$\mathcal{F}_B := \{\text{primes } p_i \leq B\}$$

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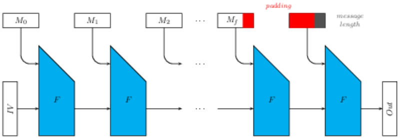
- **Linear algebra** modulo $p-1$ on the relations
- For $B \approx \exp((\log p)^{1/2})$, **subexponential complexity**

A very high-level look at SHA-1



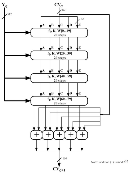


Figure 9.5 SHA-1 Processing of a Single 512-bit Block (SHA-1 Compression Function)

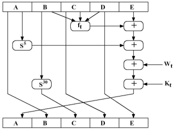


Figure 9.6 Elementary SHA Operation (single step)

- Most hash functions have a similar structure

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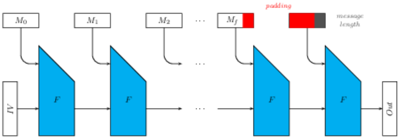

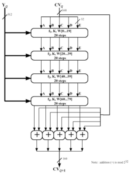

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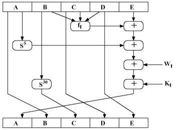

Figure 9.6 Elementary SHA Operation (single step)

- Most hash functions have a similar structure
- Security : various heuristic arguments