Small-disturbance angle stability analysis and improvement

Thierry Van Cutsem
t.vancutsem@ulg.ac.be www.montefiore.ulg.ac.be/~vct

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Examples of electromechanical oscillations

The 5-bus system (studied in previous lectures) illustrating a *local plant mode*:

- Oscillation of one machine against the rest of the system.
- Response of machine rotor speed to a 10% drop of Thévenin e.m.f., lasting 0.05 s.
- Period $\approx 1$ s.
The “Kundur” test system illustrating an *interarea oscillation*

(Almost) no power flow between left and right parts

Oscillation of machines 1 and 2 against machines 3 and 4

Response of machine rotor speeds to a 1 % increase of load at bus 9

Period $\approx 2$ s
The “Kundur” test system illustrating an interarea oscillation

oscillation of machines 1 and 2 against machines 3 and 4 with a different mode shape

400 MW power flow from left and to right part

response of machine rotor speeds to a 1% increase of load at bus 9

period $\approx 2\ s$
Classification of oscillation modes

**Local modes**: involve a small part of the system
- rotor angle oscillations of a single generator or a single plant against the rest of the system: *local plant mode*
  - can be studied using a one-machine infinite-bus system
- oscillations between rotors of a few generators close to each other: *intermachine or interplant mode oscillations*
- typical range of frequencies of local plant and interplant modes: 0.7 to 2 Hz
- may also be associated with inappropriate tuning of a control equipment (excitation system, HVDC converter, SVC, etc.): *control mode*

**Global modes**: involve large areas of the system, widespread effects
- oscillations of a large group of generators in one area swinging against a group of generators in another area: *interarea mode*
- usually, the larger the group of generators, the slower the oscillations
- typical range of frequencies of interarea modes: 0.1 to 0.7 Hz
- more complex to analyse and to damp
Objectives of this lecture

- Present an approach to analyse the small-disturbance stability of a system described by an algebraic-differential model.
- Apply this approach to a simple one-machine infinite-bus system (typical of a local plant mode study):
  - Derive the model.
  - Compute (one of) its equilibrium point(s).
  - Analyze the small-disturbance stability of this equilibrium.
- Present a method to improve small-disturbance stability by correcting troublesome eigenvalues.
- Apply this method to the design of a “power system stabilizer” acting on the one-machine infinite-bus system.
- Practice this with Matlab.
Model of a one-machine infinite-bus system

Simplifying assumptions

- $d$ axis: only the field winding is considered
- $q$ axis: only one winding ($q_1$) is considered, to simulate a damper
- the stator resistance is neglected
- saturation is neglected
- mechanical torque $T_m$ is considered constant
- rotor speed remains close to nominal value: $\omega \simeq 1$ pu
- a very simple Automatic Voltage Regulator (AVR) model is considered:
Network equations under the phasor approximation

Phasor of voltage at infinite bus: zero phase angle.

\((x, y)\) reference axes: rotate at nominal angular speed \(\omega_N\), and the axis \(x\) coincides with the rotating vector relative to the infinite bus voltage.

\[
\vec{V} = V_\infty \angle 0 + jX_e \vec{I} \quad \Leftrightarrow \quad v_x + jv_y = V_\infty + jX_e (i_x + j i_y)
\]

Decomposing in real and imaginary parts:

\[
v_x = V_\infty - X_e i_y \quad (1)
\]

\[
v_y = X_e i_x \quad (2)
\]

All variables and parameters are in per unit.
Park equations of synchronous machine under the phasor approximation

\[
\begin{align*}
\psi_d &= L_{dd} i_d + L_{df} i_f \\
\psi_q &= L_{qq} i_q + L_{qq1} i_{q1} \\
\psi_f &= L_{ff} i_f + L_{df} i_d \\
\psi_{q1} &= L_{qq1} i_{q1} + L_{qq} i_q \\
\frac{1}{\omega_N} \frac{d}{dt} \psi_f &= K v_f - R_f i_f \\
\frac{1}{\omega_N} \frac{d}{dt} \psi_{q1} &= -R_{q1} i_{q1} \\
v_d &= -\omega \psi_q = -\psi_q \\
v_q &= \omega \psi_d = \psi_d \\
2H \frac{d}{dt} \omega &= T_m - T_e = T_m - (\psi_d i_q - \psi_q i_d) \\
\frac{1}{\omega_N} \frac{d}{dt} \delta &= \omega - 1
\end{align*}
\]

- All variables are in per unit, except \( \delta \) which is in rad and \( t \) in seconds. Hence, the factor \( t_B = 1/\omega_N \) in Eqs. (7, 8, 12), where \( \omega_N \) is in rad/s.
- All parameters are in per unit, except \( H \) which is in seconds.
- \( v_f \) is in per unit on the AVR voltage base. \( K \) is a factor to pass from the AVR base to the machine base, which is used in Eq. (7).
**Change of reference axes:** from machine \((d, q)\) to system \((x, y)\) reference

In this system: \(c = 0\) and \(\omega_{ref} = \omega_N\)

\[
\begin{align*}
\nu_q + j\nu_d &= e^{-j\delta}(\nu_x + j\nu_y) = (\cos\delta - j\sin\delta)(\nu_x + j\nu_y)
\end{align*}
\]

Decomposing into real and imaginary components:

\[
\begin{align*}
\nu_d &= -\sin\delta \nu_x + \cos\delta \nu_y \quad (13) \\
\nu_q &= \cos\delta \nu_x + \sin\delta \nu_y \quad (14)
\end{align*}
\]

Similarly for the current:

\[
\begin{align*}
i_d &= -\sin\delta i_x + \cos\delta i_y \quad (15) \\
i_q &= \cos\delta i_x + \sin\delta i_y \quad (16)
\end{align*}
\]
Automatic voltage regulator

\[
\frac{d v_f}{dt} = -v_f + G(V_o - V) = \frac{-v_f + G(V_o - \sqrt{v_x^2 + v_y^2})}{T} \quad \text{(17)}
\]

- \( t \) and \( T \) are in seconds
- \( V_o, V, v_x \) and \( v_y \) are in per unit on the network base voltage
- \( v_f \) is in per unit on the AVR voltage base \( V_{fB} \)
- commonly used base: \( V_{fB} = \) field voltage that produces \( V = 1 \) pu at the terminal of the machine rotating at nominal speed with stator open.

\[
i_d = i_q = 0 \quad \Rightarrow \quad \psi_d = L_{df} i_f \quad \text{and} \quad \psi_q = 0 \quad \Rightarrow \quad v_q = L_{df} i_f \quad \text{and} \quad v_d = 0
\]

\[
\Rightarrow \quad V = 1 = \sqrt{v_d^2 + v_q^2} = L_{df} i_f \quad \Rightarrow \quad i_f = \frac{1}{L_{df}}
\]

\[
\frac{d \psi_f}{dt} = 0 \quad \text{and} \quad v_f = 1 \quad \Rightarrow \quad K = R_f i_f = \frac{R_f}{L_{df}}
\]

- \( G \) is the AVR open-loop gain in pu/pu
**Variables and equations are balanced**

17 variables:
- 5 differential: $\psi_f, \psi_{q1}, \omega, \delta, v_f$
- 12 algebraic: $v_x, v_y, i_x, i_y, v_d, v_q, i_d, i_q, i_f, i_{q1}, \psi_d, \psi_q$

17 equations:
- 5 differential: Eqs. (7, 8, 11, 12, 17)
- 12 algebraic: Eqs. (1, 2, 3, 4, 5, 6, 9, 10, 13, 14, 15, 16)

**Comments**
- In this simple system, some or all algebraic variables (and an equal number of algebraic equations) could be eliminated, thus yielding a smaller model;
- however, the techniques shown hereafter do not require performing such manipulations;
- on the contrary, keeping them in the model allow us to illustrate how differential-algebraic models are treated in practice.
Small-disturbance stability analysis

Linearization of a system described by differential equations

Consider a system described by the differential equations:

\[
\dot{x} = f(x) \quad \text{dim } x = \text{dim } f = n
\]

Let \( x^* \) be an equilibrium point : \( f(x^*) = 0 \)

Consider an infinitesimal variation of \( x \) around \( x^* \): \( \Delta x = x - x^* \)

The dynamics of \( \Delta x \) is given by:

\[
\Delta \dot{x} = \dot{x} = f(x) \simeq f(x^*) + \left. \frac{\partial f}{\partial x} \right|_{x^*} (x - x^*) = \left. \frac{\partial f}{\partial x} \right|_{x^*} \Delta x
\]

where \( \frac{\partial f}{\partial x} \) is the Jacobian of \( f \) with respect to \( x \):

\[
\frac{\partial f}{\partial x} = \begin{bmatrix} \frac{\partial f_i}{\partial x_j} \end{bmatrix} \quad i, j = 1, \ldots, n
\]

\( \left. \frac{\partial f}{\partial x} \right|_{x^*} \) is the state matrix of the linearized system.
**Stability of the equilibrium** $x^*$

Assessed from the $n$ eigenvalues of $\frac{\partial f}{\partial x}(x^*)$:

- if all eigenvalues have negative real parts, $x^*$ is stable; $x^*$ is a *sink* or *stable node*

- if at least one eigenvalue has a positive real part, $x^*$ is unstable
  - all eigenvalues with positive real part: $x^*$ is a *source* or *unstable node*
  - some eigenvalues with positive real part: $x^*$ is a *saddle*.

- if the eigenvalues have negative real parts, except some of them which have a zero real part, stability cannot be decided; higher-order terms of the Taylor series expansion have to be investigated.

In practice, to have some “margin” with respect to instability, the eigenvalues must be “at some distance” from the right half complex plane.
**Linearization of a system described by differential-algebraic equations**

Consider a system described by the algebraic-differential equations:

\[
\begin{align*}
\dot{x} &= f(x, y) \quad \text{dim } x = \text{dim } f = n \\
0 &= g(x, y) \quad \text{dim } g = \text{dim } y = m
\end{align*}
\]  

Let \((x^*, y^*)\) be an equilibrium point:

\[
\begin{align*}
0 &= f(x^*, y^*) \\
0 &= g(x^*, y^*)
\end{align*}
\]

**Implicit function theorem.** Let \(\frac{\partial g}{\partial y}\) be the Jacobian of \(g\) with respect to \(y\). At a point \((x, y)\) where \(\frac{\partial g}{\partial y}\) is nonsingular, there exists a unique and differentiable function \(\varphi\) such that locally:

\[
y = \varphi(x)
\]

Then, substituting \(\varphi(x)\) to \(y\) in (18) yields the differential (only) model:

\[
\dot{x} = f(x, \varphi(x)) = F(x)
\]

Stability has to be studied on the \((n \times n)\) Jacobian of \(F\) with respect to \(x\)!
Except in simple cases, the analytical expression of $\varphi$ cannot be derived instead, the differential-algebraic model is linearized and the algebraic states are eliminated, as shown next.

With the same notation as in the previous slides:

$$\begin{align*}
\dot{\Delta x} &= \left(\frac{\partial f}{\partial x}\right)_{x=x^*,y=y^*} \Delta x + \left(\frac{\partial f}{\partial y}\right)_{x=x^*,y=y^*} \Delta y \\
0 &= \left(\frac{\partial g}{\partial x}\right)_{x=x^*,y=y^*} \Delta x + \left(\frac{\partial g}{\partial y}\right)_{x=x^*,y=y^*} \Delta y
\end{align*}$$

(20)  (21)

Assuming that $\left(\frac{\partial g}{\partial y}\right)_{x=x^*,y=y^*}$ is nonsingular$^1$:

$$\Delta y = - \left(\frac{\partial g}{\partial y}\right)^{-1} \frac{\partial g}{\partial x} \Delta x \quad \Rightarrow \quad \dot{\Delta x} = \left[\frac{\partial f}{\partial x} - \frac{\partial f}{\partial y} \left(\frac{\partial g}{\partial y}\right)^{-1} \frac{\partial g}{\partial x}\right] \Delta x$$

Stability is analyzed from the eigenvalues of the reduced Jacobian $A$.

$^1$the dependency on the linearization point is omitted for simplicity of notation
What if $\frac{\partial g}{\partial y}$ is singular?

A point $(x, y)$ where $\frac{\partial g}{\partial y}$ is singular is called a *singularity*.

- For a small variation of $x$, the variation of $y$ becomes infinitely large
- the system dynamics becomes undefined; numerical integration cannot proceed
- the mathematical model stops matching the physical system (whose time evolution cannot stop!)
- singularities often originate from model simplifications: some dynamics assumed infinitely fast and replaced by algebraic equilibrium conditions.

**Extension to model with inputs and outputs**

Consider a system described by:

\[
\begin{align*}
\dot{x} &= f(x, y, u) \\
0 &= g(x, y, u) \\
z &= h(x, y, u)
\end{align*}
\]

where $u$ is a vector of *inputs (or controls)* and $z$ of *outputs (or measurements)*.
After linearization:

\[
\dot{\Delta x} = \frac{\partial f}{\partial x} \Delta x + \frac{\partial f}{\partial y} \Delta y + \frac{\partial f}{\partial u} \Delta u \tag{25}
\]

\[
0 = \frac{\partial g}{\partial x} \Delta x + \frac{\partial g}{\partial y} \Delta y + \frac{\partial g}{\partial u} \Delta u \tag{26}
\]

\[
\Delta z = \frac{\partial h}{\partial x} \Delta x + \frac{\partial h}{\partial y} \Delta y + \frac{\partial h}{\partial u} \Delta u \tag{27}
\]

Extracting \(\Delta y\) from (26) and substituting in (25,27):

\[
\dot{\Delta x} = \left[ \frac{\partial f}{\partial x} - \frac{\partial f}{\partial y} \left( \frac{\partial g}{\partial y} \right)^{-1} \frac{\partial g}{\partial x} \right] \Delta x + \left[ \frac{\partial f}{\partial u} - \frac{\partial f}{\partial y} \left( \frac{\partial g}{\partial y} \right)^{-1} \frac{\partial g}{\partial u} \right] \Delta u \tag{28}
\]

\[
\Delta z = \left[ \frac{\partial h}{\partial x} - \frac{\partial h}{\partial y} \left( \frac{\partial g}{\partial y} \right)^{-1} \frac{\partial g}{\partial x} \right] \Delta x + \left[ \frac{\partial h}{\partial u} - \frac{\partial h}{\partial y} \left( \frac{\partial g}{\partial y} \right)^{-1} \frac{\partial g}{\partial u} \right] \Delta u \tag{29}
\]

Standard form of a linear system with inputs and outputs.
Exercises using MATLAB

System Data

All values in per unit refer to the nominal apparent power of the machine

\[ X_e = 0.45 \text{ pu} \quad f_N = 50 \text{ Hz} \]

Synchronous machine:

\[ X_\ell = 0.2 \quad X_d = 2.2 \quad X'_d = 0.3 \quad X_q = 2.2 \text{ pu} \quad X'_q = 0.25 \text{ pu} \]

\[ T'_{do} = 7.0 \text{ s} \quad T'_qo = 0.4 \text{ s} \quad H = 4 \text{ s} \]

Automatic Voltage Regulator:

\[ T = 0.4 \text{ s} \quad G = 70 \text{ pu/pu} \]

Operating point: voltage, active and reactive powers of machine specified

\[ V = 1 \text{ pu} \quad P = 0.7 \text{ pu} \quad Q = 0.15 \text{ pu} \]

\(^2X_\ell\) is the leakage reactance
Deriving the inductances and resistances of the model  \(^{(EMFL\ pu\ system)}\)

\[
L_{dd} = X_d = 2.2 \quad L_{qq} = X_q = 2.2 \pu
\]
\[
t_B = \frac{1}{\omega_N} = \frac{1}{2\pi f_N} = 3.183 \times 10^{-3} \, \text{s}
\]
\[
T'_{do\ pu} = \frac{T'_{do\ s}}{t_B} = 2199 \pu \quad T'_{qo\ pu} = \frac{T'_{qo\ s}}{t_B} = 125.7 \pu
\]
\[
L_{df} = L_{dd} - X_\ell = 2.0 \quad L_{qq1} = L_{qq} - X_\ell = 2.0 \pu
\]
\[
X'_d = L'_d = L_{dd} - \frac{L^2_{df}}{L_{ff}} \quad \Rightarrow \quad L_{ff} = \frac{L^2_{df}}{L_{dd} - X'_d} = 2.105 \pu
\]
\[
X'_q = L'_q = L_{qq} - \frac{L^2_{qq1}}{L_{qq1q1}} \quad \Rightarrow \quad L_{qq1q1} = \frac{L^2_{qq1}}{L_{qq} - X'_q} = 2.051 \pu
\]
\[
T'_{do} = \frac{L_{ff}}{R_f} \quad \Rightarrow \quad R_f = \frac{L_{ff}}{T'_{do}} = 9.573 \times 10^{-4} \pu
\]
\[
T'_{qo} = \frac{L_{qq1q1}}{R_{q1}} \quad \Rightarrow \quad R_{q1} = \frac{L_{qq1q1}}{T'_{qo}} = 1.632 \times 10^{-4} \pu
\]
\[
K = \frac{R_f}{L_{df}} = 4.787 \times 10^{-4} \pu
\]
Values of the differential and algebraic states at the operating point

\[
\begin{align*}
V_\infty &= 0.9843 \quad v_x = 0.9474 \quad v_y = 0.3200 \text{ pu} \\
i_x &= 0.7112 \quad i_y = 0.0819 \text{ pu} \quad \delta = 1.1842 \text{ rad} \\
i_d &= -0.6278 \quad i_q = 0.3440 \quad v_d = -0.7568 \quad v_q = 0.6536 \text{ pu} \\
\psi_d &= 0.6536 \quad \psi_q = 0.7568 \quad \psi_f = 0.8863 \quad \psi_{q1} = 0.6880 \text{ pu} \\
i_f &= 1.0174 \quad i_{q1} = 0 \quad v_f = 2.0348 \text{ pu}
\end{align*}
\]

Short exercises

- Check that these values of \(v_x, v_y, i_x\) and \(i_y\) yield \(P = 0.7\) and \(Q = 0.15\) pu
- same question using \(v_d, v_q, i_d\) and \(i_q\)
- compute the electromagnetic torque \(T_e\) in pu. Comment on its value
- compute the magnitude of the e.m.f \(E_q\) behind synchronous reactances (i) from the value of \(i_f\); (ii) from the equation: \(\bar{E}_q = \bar{V} + jX_d\bar{i}_d + jX_q\bar{i}_q\)
The Matlab script `omib.m`

- derives the inductances and resistances of the model as shown in slide # 20
- computes the state variables at the operating point as shown in slide # 21
- computes the “full” Jacobian:

\[
J = \begin{bmatrix}
\frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} & \frac{\partial f}{\partial u} \\
\frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} & \frac{\partial g}{\partial u} \\
\frac{\partial h}{\partial x} & \frac{\partial h}{\partial y} & \frac{\partial h}{\partial u}
\end{bmatrix}
\]

for the particular case:

\[
z = \omega \quad u = V_o
\]

- computes the matrices A, B, C and D of the linearized system
Exercise # 1

- Compute the system eigenvalues assuming: constant flux $\psi_f$, constant flux $\psi_{q1}$ and constant voltage $v_f$
- compute the system eigenvalues assuming: constant flux $\psi_{q1}$ and constant voltage $v_f$. Comment on the influence of the field winding
- compute the system eigenvalues assuming constant voltage $v_f$. Comment on the influence of the $q1$ (damper) winding
- compute the system eigenvalues under AVR control
- compare the period of electromechanical oscillations in all four cases.

Exercise # 2

- How do the system eigenvalues evolve when increasing the active power to $P = 0.9$ pu (leaving $Q$ and $V$ unchanged) ?
- How do the system eigenvalues evolve when increasing the gain $G$ from 70 to 120 (keeping $T = 0.4$ s) ?
- How do the system eigenvalues evolve when decreasing the time constant $T$ from 0.4 to 0.1 s (keeping $G = 70$) ?
**Exercise # 3**

With $P = 0.9$ pu and $V = 1$ pu, determine, with an accuracy of 0.05 pu, the maximum reactive power that the generator:

- can produce
- can absorb

without the operating point becoming unstable, under AVR control.

Which eigenvalues become unstable?
Left and right eigenvectors

Consider an $n \times n$ matrix $A$ with all distinct and nonzero eigenvalues $\lambda_i$.

Let $v_i$ be the right eigenvector$^3$ of $\lambda_i$:

$$Av_i = \lambda_i v_i \quad (i = 1, \ldots, n)$$

and $w_i$ the left eigenvector of $\lambda_i$:

$$w_i^T A = \lambda_i w_i^T \iff A^T w_i = \lambda_i w_i \quad (i = 1, \ldots, n)$$

Let $V$ (resp. $W$) be the matrix of right (resp. left) eigenvectors:

$$V = \begin{bmatrix} v_1 & \cdots & v_n \end{bmatrix} \quad W = \begin{bmatrix} w_1^T \\ \vdots \\ w_n^T \end{bmatrix}$$

It is easily shown that: $W = V^{-1}$ and $WAV = \text{diag}(\lambda_i)$

$^3$notation: all vectors are column vectors
Controllability and observability of a mode

Consider a system with state vector $\mathbf{x}$, a scalar input $u$ and a scalar output $y$:

$$\dot{\mathbf{x}} = A\mathbf{x} + \mathbf{b}u \quad z = c^T\mathbf{x} + du$$

Consider the change of variables: $\tilde{\mathbf{x}} = W\mathbf{x}$.

$$W^{-1}\dot{\tilde{\mathbf{x}}} = AW^{-1}\tilde{\mathbf{x}} + \mathbf{b}u \quad z = c^TW^{-1}\tilde{\mathbf{x}} + du$$

$$\dot{\tilde{\mathbf{x}}} = WAW^{-1}\tilde{\mathbf{x}} + Wb\mathbf{u} = \Lambda\tilde{\mathbf{x}} + W\mathbf{b}\mathbf{u} \quad z = c^TV\tilde{\mathbf{x}} + du$$

where $\Lambda = \text{diag}(\lambda_i)$

For the $i$-th “mode” $\lambda_i$:

- the larger $(W\mathbf{b})_i = \mathbf{w}_i^T\mathbf{b}$, the more the mode can be controlled by $u$
- the larger $(c^TV)_i = c^TV_i$, the more the mode can be observed in $z$. 
Transfer function and residues

\[ F(s) = \frac{Z(s)}{U(s)} = c^T V (sI - \Lambda)^{-1} Wb + d \]

\[ = \begin{bmatrix} c^T v_1 & \ldots & c^T v_n \end{bmatrix} \text{diag} \left( \frac{1}{s - \lambda_i} \right) \begin{bmatrix} w_1^T b \\ \vdots \\ w_n^T b \end{bmatrix} + d \]

\[ = \sum_{i=1}^{n} \frac{c^T v_i w_i^T b}{s - \lambda_i} + d = \sum_{i=1}^{n} \frac{R_i}{s - \lambda_i} + d \]

The residue \( R_i \) relative to the \( i \)-th mode \( \lambda_i \):

- depends on both the observability and the controllability of \( \lambda_i \)
- is even smaller than \( F(s) \) has a zero \( \zeta_k \) close to \( \lambda_i \). Indeed:

\[ R_i = \lim_{s \to \lambda_i} (s - \lambda_i)F(s) = \lim_{s \to \lambda_i} (s - \lambda_i) \frac{\prod_{k=1}^{m}(s - \zeta_k)}{\prod_{j=1}^{n}(s - \lambda_j)} = \lim_{s \to \lambda_i} \frac{\prod_{k=1}^{m}(s - \zeta_k)}{\prod_{j=1,j \neq i}^{n}(s - \lambda_j)} \]

- would be zero in case of exact zero-pole cancellation.
Synthesis of a stabilizing feedback using residues

Consider a compensator using $z$ as input and acting on $u$

Which condition should be satisfied by the transfer function $G(s)$ in order to stabilize the mode $\lambda_c$ of the uncompensated system?

The closed-loop transfer function is

$$\frac{F(s)}{1 - KF(s)G(s)}$$

Let $\tilde{s}$ be one of the closed-loop poles:

$$1 - KF(\tilde{s})G(\tilde{s}) = 0$$

$$1 - K \left[ \sum_{i} \frac{R_i}{\tilde{s} - \lambda_i} + d \right] G(\tilde{s}) = 0$$

$$1 - K \sum_{i \neq c} \frac{R_i}{\tilde{s} - \lambda_i} G(\tilde{s}) - K \frac{R_c}{\tilde{s} - \lambda_c} G(\tilde{s}) - K d G(\tilde{s}) = 0$$ (30)
Consider a closed-loop pole $\tilde{s}$ lying on the branch of the root locus which starts from the open-loop pole $\lambda_c$.

When the compensator gain $K$ tends to zero, $\tilde{s}$ tends to $\lambda_c$.

Keeping the dominant terms only in (30):

$$1 - R_c \ G(\lambda_c) \ \lim_{K \to 0} \frac{K}{\tilde{s} - \lambda_c} = 0 \quad \text{or} \quad \lim_{K \to 0} \frac{\tilde{s} - \lambda_c}{K} = R_c \ G(\lambda_c)$$

In the complex plane $\lim_{K \to 0} \frac{\tilde{s} - \lambda_c}{K}$ is a vector tangent to the branch of the root locus starting from $\lambda_c$.

In order to shift the eigenvalue $\lambda_c$ to the left:

- the branch of the root locus should leave $\lambda_c$ at an angle of 180 degrees
- $R_c \ G(\lambda_c)$ should be a real negative number
- $G(s)$ must be such that $\angle G(\lambda_c) = \pm 180^\circ - \angle R_c$
Improvement of small-disturbance angle stability

Principle:
- increase the damping torques of synchronous machines
- move the complex eigenvalues corresponding to the unstable or badly damped mode into the desired region of the complex plane.

How and where?
- add a power system stabilizer acting through the Automatic Voltage Regulator (AVR): the less expensive solution
- take advantage of the presence of power electronics-base components to vary
  - the shunt susceptance of a Static Var Compensator (SVC)
  - the active power flowing through a High Voltage Direct Current (HVDC) link
  - the series reactance of a Thyristor Controlled Series Capacitor (TCSC)
  - another Flexible AC Transmission System (FACTS) device.
Let $\lambda_c$ be the badly-damped/unstable electromechanical mode.

Since $\lambda_c$ is close to the imaginary axis: $\lambda_c \simeq j \text{imag}(\lambda_c) = j \omega_c$

The PSS increases the damping torque in a range of frequencies around $\omega_c$. 
The PSS transfer function decomposes into:

\[ K_{pss} G(s) = K_{pss} G_1(s) G_2(s) G_3(s) \]

Transfer function \( G_1(s) \):
- shifts \( \lambda_c \) to the left in the complex plane by bringing a phase compensation according to the residue method:
  \[ \angle G_1(\lambda_c) \simeq \angle G_1(j \omega_c) = \pm 180^\circ - \angle R_c \]
- \( G_1(s) \) corresponds to one or several lead-lag filters: see slide \# 35
- the latter are “tuned” to provide their maximum phase shift \( \phi_m \) at the frequency \( \omega_c \)
Transfer function $G_2(s)$:

- In steady state and for slow variations, the PSS must not affect voltage regulation.
- $G_2(s)$ is a washout (or high-pass) filter: see slide #36.
- $T_w$ is taken large enough to not modify the phase angle of $G_1$ for frequencies around $\omega_c$. For instance:

$$\frac{10}{T_w} \sim \frac{\omega_c}{10}$$

Transfer function $G_3(s)$ (optional):

- In a thermal power plant, the turbine stages, the generator and the exciter are mounted on a relatively long shaft. The latter has torsional oscillation frequencies in the range $10 - 15$ Hz and higher.
- The PSS must not excite those frequencies.
- The risk is higher for a PSS using the rotor speed as input signal.
- In this case, $G_3$ is a low-pass filter so that the PSS contribution is negligible at the lowest torsional frequency and above.
Gain $K_{pss}$:

- Adjusted until the corrected mode $\tilde{\lambda}_c$ has a damping ratio:

$$\xi = \frac{-\text{real}(\tilde{\lambda}_c)}{|\tilde{\lambda}_c|} = \frac{-\text{real}(\tilde{\lambda}_c)}{\sqrt{[\text{real}(\tilde{\lambda}_c)]^2 + [\text{imag}(\tilde{\lambda}_c)]^2}}$$

higher than some value:

$$\xi \geq 0.05 \sim 0.10$$

- While $K_{pss}$ is increased, the other eigenvalues are monitored since they might move to the right (the residue method allows controlling a single mode !)

- For excessive values of $K_{pss}$, the branch of the root locus that starts from $\lambda_c$ might “bend” to the right (the residue method focuses on a neighbourhood of the mode to correct !)
Lead-lag filter

$$G(s) = \frac{1 + s\alpha \tau}{1 + s\tau}$$

$\alpha > 1$ to obtain phase lead  \hspace{1cm}  $\alpha < 1$ to obtain phase lag

Bode plot (lead filter)

Angular frequency at which the phase is maximum:  
$$\omega_m = \frac{1}{\tau \sqrt{\alpha}}$$

Maximum phase:  
$$\phi_m = \arcsin \frac{\alpha - 1}{\alpha + 1} \hspace{1cm} \Rightarrow \hspace{1cm} \alpha = \frac{1 + \sin \phi_m}{1 - \sin \phi_m}$$

To obtain $\phi_m > 60^\circ$ use two filters in cascade, etc.
Washout filter

\[ G(s) = \frac{s}{1 + sT_w} \]

Bode plot

- The phase is negligible for \( \omega > \frac{10}{T_w} \)